SIMG-712-90 FINAL EXAM Open Book

Due F 2/25/2005, 7 PM EST (unless other agreement)
No consultations with carbon-based life forms other than the instructor!
Standard Hint: make sketches before writing down equations.
State any assumptions you make. Show your work!!

Select 5 of 6 (equal weighting, even though not equal difficulty)

1. You are to design an imaging system for use in spacebased astronomy in low earth orbit – this is just to ensure that there are no atmospheric aberrations. The sensor is a pixelated CCD whose square pixels are $9\,\mu\text{m} \times 9\,\mu\text{m}$ that butt against each other so that all photons onto the sensor are imaged (there is no "dead" area between CCD pixels). The diameter of the optic is d mm and the focal length is f mm..

   (a) Determine the f/# of the system such that the diameter of the central core of the diffraction spot (the circle enclosed by the first zero of the Fraunhofer diffraction pattern) just fills a CCD pixel for $\lambda = 550\,\text{nm}$.

   (b) Determine the focal length of the system that results in an "angular resolution" (also called a "plate scale") of 1 arcsecond per pixel at the same wavelength. Use this result to determine the diameter of the optic that also satisfies the condition in part (a).

   (c) For the same diameter and focal length, determine the size of the diffraction spot and the plate scale for light with $\lambda = 400\,\text{nm}$ and $\lambda = 1000\,\text{nm}$.

2. The area density of cone sensors in the fovea of the eye is approximately $160,000\,\text{mm}^{-2}$. For imaging in bright light, assume that the diameter of the eye's pupil is approximately 2 mm.

   (a) Determine the approximate angular resolution of the eye when viewing an object at ∞. Assume that the wavelength $\lambda = 500\,\text{nm}$.

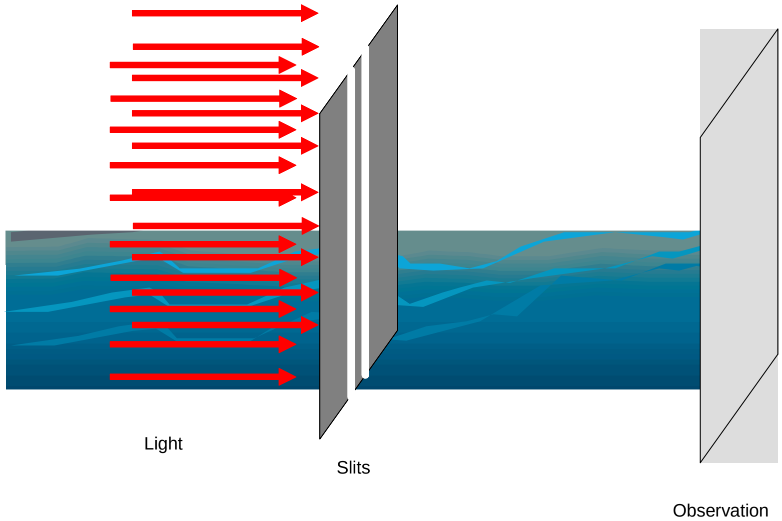
   (b) Determine if the image is sampled adequately or undersampled by the cones in the fovea (assume that the lens of the eye has no significant aberrations).

   (c) Describe in words the changes in the optics of the eye as the object distance is decreased from ∞ to the near point (the closest distance where the eye lens can focus)

   (d) This problem uses the eye model that was used to illustrate ray tracing on p. 203 of the most recent notes (dated 2/12/2005). If the distance from the near point to the eye is the "standard distance" of 10" $\cong 250\,\text{mm}$, if the distance from the cornea to the retina remains fixed at the distance for distant vision (object at ∞), and if the two surfaces of the eye lens have equal and opposite curvatures regardless of the object distance, determine the radii of curvature of the surfaces of the eye lens that are necessary to position the image on the retina.

   (e) Assuming that a page of printed text is held at the "near point," determine the smallest linear dimension that the eye can read clearly given the density of cones and the pupil diameter.

3. A double slit experiment is performed with the slits both above and under a water surface ($n = 1.33$). Suppose that the separation of the slits is 0.35 mm, while the temporal frequency of the light is $\nu = 5 \times 10^{14}$ Hz. The distance from the slits to the observation screen is 2 m.



   (a) Describe the differences (if any) between the patterns observed on the screen under water and in air.

   (b) Sketch the patterns in both media.

   (c) Determine the separation on the observation screen between the central bright fringe and the second bright fringe.

4. A binary (or *bitonal*) image of size $N \times N$ pixels contains a rectangular region of level "1" surrounded by pixels with level "0."

   (a) Determine the magnitude of the gradient of this image and the approximation obtained via the equation on bottom of p.411 (notes dated 2/12/2005):

$$|\nabla f[n,m]| = \sqrt{(\partial_x * f[n,m])^2 + (\partial_y * f[n,m])^2}$$
$$\cong |\partial_x * f[n,m]| + |\partial_y * f[n,m]|$$

   (b) Determine the histogram of edge "directions" computed from the azimuth angle of the gradient.

   (c) Determine the values of the "isotropic" 3×3 Laplacian operator.

5. A digital image $f[n,m]$ is created from random noise, i.e., random variations in gray level. There are two such areas in the image that have the same mean gray values but different variances. Describe how to segment the two regions by applying pixel (point) operators and linear shift-invariant local neighborhood operations (convolutions). In other words, the only nonlinear operation available to you is a pixel operation. Be sure to specify the sequence of operations and the kernels used in any convolutions.

6. A digital image has 8 gray values selected from the probability distribution"

$$
\begin{aligned}
p[0] &= 0.19 \\
p[1] &= 0.25 \\
p[2] &= 0.21 \\
p[3] &= 0.16 \\
p[4] &= 0.08 \\
p[5] &= 0.06 \\
p[6] &= 0.03 \\
p[7] &= 0.02
\end{aligned}
$$

(a) Calculate the information content (also called the *entropy*) in BOTH binary digits and base-e digits (called *nats*) bits per pixel

(b) Construct a Huffman code for images created from this probability distribution.

(c) Calculate the expected bit rate (bits per pixel) for an image from this distribution compressed using this Huffman code.

(d) (OPTIONAL BONUS) How many unique Huffman codes exist for an image that consists of three different gray levels. Construct them.