1. An image has been quantized to 10 gray levels per pixel and has the following histogram data. Generate the look-up table that will equalize the histogram.

<table>
<thead>
<tr>
<th>Gray Value</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>27</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>17</td>
</tr>
<tr>
<td>8</td>
<td>32</td>
</tr>
<tr>
<td>9</td>
<td>19</td>
</tr>
</tbody>
</table>

(a) Plot the histogram
(b) Generate the look-up table that will equalize the histogram.
(c) Plot the equalized histogram

2. Consider the $4 \times 4$ quantized to 3 bits:

```
1 2 5 6
1 1 4 5
0 1 4 7
3 1 3 4
```

(a) Sketch the gray level image histogram.
(b) Characterize the histogram (one mode, bimodal, etc.). If bimodal, pick a threshold for binary thresholding transformation.
(c) Sketch the histogram of the image after the thresholding transformation.

3. Design the impulse response of a filter that calculates an average along the radial direction specified by $\phi = +\frac{3\pi}{4}$ and calculates a second derivative along the orthogonal direction. What would this filter be useful for?

4. A rectangular object is to be imaged by a system that adds sinusoidal noise. The image to be recorded is:

$$g [x, y] = RECT \left[ \frac{x}{16 \text{mm}}, \frac{y}{32 \text{mm}} \right] + \cos \left[ 2\pi \left( \frac{x}{4 \text{mm}} + \frac{y}{6 \text{mm}} \right) \right]$$

The image is to be sampled with $\Delta x = \Delta y = 1 \text{mm}.$
(a) Design the transfer function of a filter that will “block” the sinusoidal noise while transmitting as much information about the rectangle as possible.

(b) What changes do you expect to see in the box image after filtering?

5. Consider the result of a $7 \times 7$ uniform averaging filter to a digital image $N$ times (ignore any effects at the edges of the image). Characterize the expected effect.

6. Consider this $8 \times 8$ three-bit image that contains an edge and some isolated noise points:

   \[
   \begin{array}{cccccccc}
   0 & 0 & 1 & 2 & 3 & 1 & 1 & 1 \\
   0 & 0 & 1 & 2 & 3 & 1 & 1 & 1 \\
   0 & 0 & 1 & 2 & 3 & 1 & 6 & 1 \\
   0 & 0 & 1 & 2 & 3 & 1 & 1 & 1 \\
   0 & 0 & 1 & 2 & 7 & 1 & 1 & 1 \\
   0 & 0 & 1 & 2 & 3 & 1 & 4 & 1 \\
   0 & 0 & 1 & 2 & 3 & 1 & 1 & 1 \\
   0 & 0 & 1 & 2 & 3 & 1 & 1 & 1 \\
   \end{array}
   \]

   (a) Convolve this image with each of the following $3 \times 3$ convolution kernels.

   (b) Deal with the edges in a manner you think appropriate, but specify what you are doing.

   (c) Comment on the character of the output in each case.

1. $h_a[n,m] = \begin{bmatrix} -1 & -1 & -1 \\ +1 & +2 & -1 \\ +1 & +1 & -1 \end{bmatrix}$

2. $h_b[n,m] = \begin{bmatrix} -1 & -1 & -1 \\ +1 & +2 & -1 \\ +1 & +1 & -1 \end{bmatrix}$

3. $h_c[n,m] = \begin{bmatrix} +1 & +1 & +1 \\ +1 & +1 & +1 \\ +1 & +1 & +1 \end{bmatrix}$

4. $h_d[n,m] = \begin{bmatrix} 0 & 0 & 0 \\ +1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

5. $h_e[n,m] = \begin{bmatrix} +1 & +1 & +1 \\ +1 & -8 & +1 \\ +1 & +1 & +1 \end{bmatrix}$

(d) Apply a $3 \times 3$ median filter to the digital image. Characterize the output relative to those in (c)