1. Consider a planar glass plate (i.e., the faces of the plate are parallel) of index $n$ immersed in air

(a) Show that a ray incident at angle $\theta$ to the surface will emerge from the plate at the same angle.

(b) If the thickness of the plate is $d$ units, derive an expression for the physical displacement $a$ of the emerging ray relative to the original ray as a function of the incident angle.

(c) Imagine a stratified system consisting of planar layers of transparent materials of different thicknesses. Show that the propagation direction of the emerging beam is determined by only the incident direction and the refractive indices of the initial and final layers ($n_1$ and $n_f$).

2. Three lenses with focal lengths $f_1 = +100\,\text{mm}$, $f_2 = -100\,\text{mm}$, and $f_3 = +100\,\text{mm}$ are placed in that order and each is separated from the next by $t_n = 20\,\text{mm}$.

(a) Determine the focal length of the system.

(b) Locate the principal and focal points.

3. Prove that if the principal points of a biconvex lens of thickness $d$ in vacuum overlap midway between the vertices, then the lens is a sphere.

4. The focal length of a biconvex thin lens made of glass with $n = 1.5$ is known to be $500\,\text{mm}$ if measured in air. When immersed in a transparent liquid, the focal length is measured to be half as long. Determine the refractive index of the liquid.

5. Assume that the refractive index of a plano-convex lens is $n = 1.5$ and that the thickness is $6\,\text{mm}$. The radius of curvature of the convex surface is $25\,\text{mm}$ and is positioned “forward” (towards the light from the object). Derive the system matrix (the “vertex-to-vertex matrix”) for this lens and use it to determine the focal length and to locate the principal and focal points.

6. An object of height $20\,\text{mm}$ is imaged by a reflective spherical ball bearing whose diameter is $25\,\text{mm}$. The object is $500\,\text{mm}$ from the front vertex of the bearing. Locate the image, describe its character (real or virtual) and determine its height.

7. Light of a single wavelength $\lambda_0$ illuminates two small apertures in an opaque screen (the apertures can be considered to be points) located at $[x, y] = [\pm \frac{d}{2}, 0]$. The light travels down the $z$-axis a distance $L$ where in encounters a screen. The pattern of irradiance on the screen is sinusoidal fringes that vary along the $x$-axis. Determine the period of the fringes (from maximum to maximum) as a function of $L, \lambda_0$, and $d$. 

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