SIMG-712-90-20042 Homework #1
Due Th, 12/9/2004

NOTE: These are "review" problems intended to encourage reading of the review material and setting the stage for more difficult problems to come...

1. A source of harmonic motion of the form $y(t) = 6 \cdot \cos(\omega_0 t)$ located at the origin of the spatial coordinate system emits a wave that travels through a uniform (i.e., homogeneous) medium at a rate of 60 mm per second.

   (a) Find the formula for the displacement due to this wave at a distance of 800 mm from the origin.

   (b) Find the displacement at that distance for $t = 60$ s.
   (HINT: It is ALWAYS useful to draw a diagram of the problem before trying to solve it!)

2. What is the phase difference (in radians) between any two points on a harmonic electromagnetic wave separated by $\Delta z = 1\ \mu$m (1 micron) if the wavelength is $\lambda = 550$ nm?

3. N simple temporal harmonic oscillatory motions ($N > 1$) with the same amplitude and temporal frequency are superimposed (summed), i.e., the output $g[t]$ may be written as:

$$g[t] = \sum_{n=1}^{N} A_0 \ \cos[2\pi\nu_0 t + \phi_n]$$

   The phase difference between successive pairs of oscillations is:

$$\Delta\phi_n = \phi_n - \phi_{n-1},\ \textit{where}\ n = 2, 3, \cdots, N$$

   If the phase difference between each successive pair is invariant, find one value of this phase difference as a function of N for which the amplitude of the sum is zero. Again, it may be useful to draw a picture to help you solve the problem, and it also may be useful to consider the result for small integer values of N.

4. Use the Euler relation: $e^{i\theta} = \cos[\theta] + i\ \sin[\theta]$ to derive expressions for the following in terms of $\cos[\theta_1]$, $\sin[\theta_1]$, $\cos[\theta_2]$, and $\sin[\theta_2]$:

   (a) $\sin[2\theta_1]$

   (b) $\cos[\theta_1 \pm \theta_2]$

   (c) $\sin[\theta_1 \pm \theta_2]$

5. Use the formulas for $\cos[\theta_1 \pm \theta_2]$ and for $\sin[\theta_1 \pm \theta_2]$ just derived to derive expressions for the following in terms of the sum and difference frequencies $\omega_1 \pm \omega_2$. ALSO plot the results for $\omega_1 = 1$ and $\omega_2 = \frac{3}{2}$ radian per second, respectively.

   (a) $\sin[\omega_1 t] \cdot \sin[\omega_2 t]$

(b) $\cos[\omega_1 t] \cdot \cos[\omega_2 t]$

(c) $\sin[\omega_1 t] \pm \sin[\omega_2 t]$

(d) $\cos[\omega_1 t] \pm \cos[\omega_2 t]$

6. Consider the superposition of two sinusoidal traveling waves:

$$\begin{aligned} f_1[z,t] &= A_1 \cos[k_1 z - \omega_1 t], \\ A_1 &= 10\,\text{mm},\ \nu_1 = 1000\,\text{Hz},\ \text{v}_1 = 250\frac{\text{m}}{\text{s}} \\ f_2[z,t] &= A_2 \cos[k_2 z - \omega_2 t], \\ A_2 &= 9\,\text{mm},\ \nu_2 = 1500\,\text{Hz},\ \text{v}_2 = 500\frac{\text{m}}{\text{s}} \end{aligned}$$

(a) Find an expression for the resulting wave in terms of the average wave, the modulation wave, plus any remaining amplitude.

(b) Calculate the wavelengths of the average and modulation waves.

(c) Find the velocities of the average and modulation waves.

(d) Does this system exhibit normal or anomalous dispersion?

7. The phase velocity of waves in some medium is proportional to $\omega^{+\frac{1}{4}}$. Find an expression for the modulation velocity and determine whether the waves exhibit normal or anomalous dispersion.