

SIMG-712-01-20042      Solution Set #5

1. Light of a single wavelength  $\lambda_0$  illuminates two small apertures in an opaque screen (the apertures can be considered to be points) located at  $[x, y] = [\pm \frac{d}{2}, 0]$ . The light travels down the  $z$ -axis a distance  $L$  where it encounters a screen. The pattern of irradiance on the screen is sinusoidal fringes that vary along the  $x$ -axis. Determine the period of the fringes (from maximum to maximum) as a function of  $L$ ,  $\lambda_0$ , and  $d$ . *From the notes,*

$$\begin{aligned}
 D_{\text{mod}} &= \frac{\lambda}{\sin[\theta]} \\
 \text{but } \sin[\theta] &\cong \frac{d}{L} \implies \boxed{D_{\text{mod}} \cong \left(\frac{\lambda}{\frac{d}{L}}\right)} \\
 &\implies d \cdot D_{\text{mod}} \cong L \cdot \lambda
 \end{aligned}$$

2. A particular optical system has an aperture shape that is a rectangle with sides of length  $d_1$  and  $d_2$ , given by:

$$A[x, y] = \begin{cases} 1 & \text{if } -\frac{d_1}{2} \leq x \leq +\frac{d_1}{2} \text{ and } -\frac{d_2}{2} \leq y \leq +\frac{d_2}{2} \\ 0 & \text{otherwise} \end{cases}$$

Calculate the irradiance pattern formed by the system imaging a point source at infinity in the Fraunhofer diffraction limit. (Don't worry about the overall scale or normalization, but please do calculate the Fourier integral explicitly.)

*The aperture clearly can be written in the form:*

$$A[x, y] = \text{RECT} \left[ \frac{x}{d_1}, \frac{y}{d_2} \right]$$

*The wavefronts from a point source at infinity are planar, so the amplitude at the aperture is:*

$$E[x, y, z = 0] = E_0 \cdot A[x, y] = E_0 \cdot \text{RECT} \left[ \frac{x}{d_1}, \frac{y}{d_2} \right]$$

*The irradiance in the image plane is proportional to the squared magnitude of the scaled Fourier transform of the aperture function. The steps in the process:*

$$\begin{aligned} \mathcal{F}_2 \{E_0 \cdot A[x, y]\} &= E_0 \cdot \mathcal{F}_2 \left\{ \text{RECT} \left[ \frac{x}{d_1}, \frac{y}{d_2} \right] \right\} \\ &= E_0 \cdot d_1 d_2 \cdot \text{SINC} [d_1 \xi, d_2 \eta] \end{aligned}$$

*The frequency coordinate  $\xi$  is scaled back to the space coordinate  $x$  via the proportionality constant:*

$$\xi = \frac{x}{\lambda f}$$

*where  $\lambda$  is the wavelength and  $f$  is the focal length of the imaging system, which was not specified. Thus the observed irradiance is:*

$$I[x, y] \propto \left| E_0 \cdot d_1 d_2 \cdot \text{SINC} \left[ \left( \frac{d_1}{\lambda f} \right) x, \left( \frac{d_2}{\lambda f} \right) y \right] \right|^2$$

$$I[x, y] \propto E_0^2 \cdot d_1^2 \cdot d_2^2 \cdot \text{SINC}^2 \left[ \left( \frac{d_1}{\lambda f} \right) x, \left( \frac{d_2}{\lambda f} \right) y \right]$$

3. The diameter of a telescope objective is 120 mm and the focal length is 1500 mm. Light with a mean wavelength of  $\lambda = 550$  nm from a distant star enters the telescope as a (nearly) collimated beam. Compute the *radius* of the central disk of light in the image of the star on the focal plane of the lens. Assume no aberrations from the lens or the atmosphere.

(zzzzz) *The aperture function is:*

$$a(r) = \text{CYL}\left(\frac{r}{120 \text{ mm}}\right) \implies A(\rho) = \pi \frac{(120 \text{ mm})^2}{4} \cdot \text{SOMB}(120 \text{ mm} \cdot \rho)$$

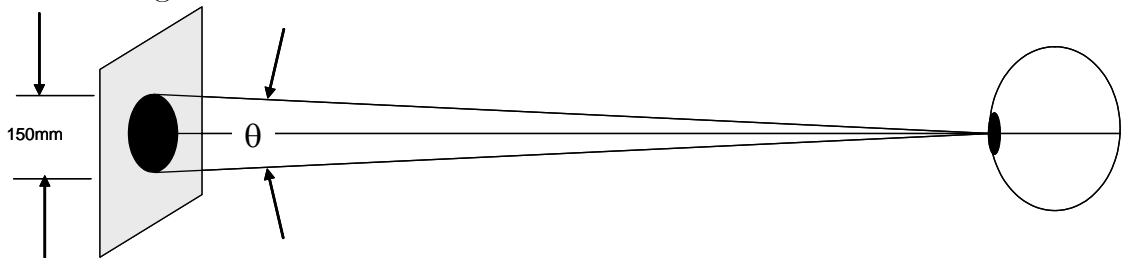
*The first zero of the sombrero function is at the first zero of  $J_1(120 \text{ mm} \cdot \rho)$ , which occurs at:*

$$\begin{aligned} 120 \text{ mm} \cdot \rho_0 &\cong 1.22 \implies \rho_0 \cong \frac{1.22}{120 \text{ mm}} \\ &= 0.0102 \text{ cycles per mm} \end{aligned}$$

*In the Fraunhofer diffraction region, the spatial frequency is scaled by the product of the wavelength and focal length:*

$$\begin{aligned} r_0 &\cong \rho_0 \cdot \lambda f \\ &\cong 0.0102 \text{ mm}^{-1} \cdot 550 \text{ nm} \cdot 1500 \text{ mm} \\ &\boxed{r_0 \cong 8.39 \mu\text{m} = 8.39 \times 10^{-3} \text{ mm}} \end{aligned}$$

4. Assuming that the eye can resolve the image of an object that subtends one arcminute, determine the distance at which a normal eye can see a black circle of diameter 150 mm on a white background.



(more zzzzz) One arcminute can should be scaled to radians:

$$\begin{aligned}
 1' &= \frac{1^\circ}{60} = \frac{\pi \text{ radians}}{180^\circ} \cdot \frac{1^\circ}{60} \\
 &\cong 2.91 \times 10^{-4} \text{ radians} \\
 &= 29.1 \text{ milliradians}
 \end{aligned}$$

The angle  $\theta$  clearly is small so it is easy to determine the distance at which 150 mm subtends 29.1 mrad.

$$\frac{150 \text{ mm}}{L} \cong 2.91 \times 10^{-4} \implies L \cong \frac{150 \text{ mm}}{2.91 \times 10^{-4}} \cong 5.15 \times 10^5 \text{ mm} = \boxed{515 \text{ m} = L}$$