

Do Problems #1 and #2 and select THREE from #3 - #6. Show your work – do not just write down results without explanation.

1. A lens system is composed of two thin lenses separated by a variable distance t . The prescriptions for the surfaces of the two lenses are:

$$L_1 : n = 1.5, R_1 = +500 \text{ mm}, R_2 = +200 \text{ mm}$$

$$L_2 : n = 1.6, R_1 = -100 \text{ mm}, R_2 = -200 \text{ mm}$$

- (a) Find the focal lengths of the two thin lenses.
 - (b) Find the focal length of the system formed from these two lenses in contact.
 - (c) Characterize the image of an object created by the system of the two lenses in contact. The object is 20 mm tall and 2 mm “deep” (dimension along the direction of the optical axis). If the depth “midpoint” of the object is \mathbf{O} , then the object distance is $\overline{\mathbf{OV}} = 250 \text{ mm}$.
 - (d) Find the separation t such that the power of the lens system is 0 diopters. Sketch the system with this separation t , showing the path traveled by a ray entering the system parallel to the optical axis (i.e., from an object an infinite distance away).
2. A reflective sphere (imagine a ball bearing) of diameter $d = 50 \text{ mm}$ acts as a spherical mirror that can be used to image objects.
- (a) Determine the focal length of the imaging “system” composed of this sphere.
 - (b) Sketch the “system,” including the location of the *image-space* focal and principal points (no need to do the object-space focal and principal points, but you may if you wish).
 - (c) Determine the location of the input object that produces a paraxial image at the center of the sphere.
 - (d) Determine the location of the input object that produces a paraxial image at the vertex of the mirror.
 - (e) Determine the transverse magnification for the object-image combination in part (d).
 - (f) Sketch the object, system, and image in the configuration in part (d).

MORE→→→

Select THREE of the Following:

3. A mercury thermometer is constructed from a cylindrical glass tube ($n = 1.5$). The outer diameter is $\frac{3}{2}$ larger than the inner diameter of the tube. The outer diameter is small (a few mm) and *much* smaller than the viewing distance; this means that the rays reaching the eye are approximately parallel. Determine the apparent diameter of the mercury column (i.e., the diameter of the inner wall of the glass tube) relative to the apparent outside diameter. HINT: sketch the entire “system” first.
4. The magnitude of the electric field for spherical electromagnetic waves emanating from a point source at the origin may be written

$$E [r, t] = \frac{E_0}{r} \cos [k_0 r - \omega_0 t]$$

where E_0 is a constant and r is the radial coordinate. With a short calculation based on the definition of the Poynting vector, show that E MUST vary as r^{-1} .

5. Regardless of the polarization, the *reflectance* of a material at an interface is just the square of the amplitude reflectance coefficient, but the *transmission* at the interface includes an additional multiplicative factor than just the square of the amplitude transmittance coefficient.
 - (a) Explain why this is so and illustrate your answer with sketches
 - (b) Derive the additional multiplicative factor.
6. The index of refraction can be approximately represented by Cauchy’s equation:

$$n \cong A + \frac{B}{\lambda_0^2}$$

where λ_0 is the wavelength in vacuum. For a particular material at $\lambda_0 = 500$ nm, the coefficients are:

$$\begin{aligned} A &= 1.5 \\ B &= 3 \cdot 10^4 \text{ nm}^2 \end{aligned}$$

- (a) Find the phase and modulation (“group”) velocities of light in this material at this wavelength.
- (b) Determine if the material exhibits normal or anomalous dispersion