

SIMG-712-01 FINAL EXAM Two Hours

There are 9 problems with point values listed.

Select Problems whose values equal or exceed 100%: your score will be the ratio of points received to points attempted (note optional extra-credit question at end!)

Standard Hint: make sketches before writing down equations.

Show your work!! State any assumptions you make.

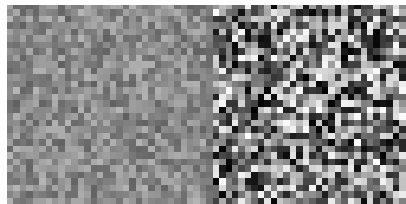
Possibly useful expressions listed at end.

First impressions can deceive! If the problem looks difficult, it may not be; if it looks easy, be careful!

1. (25%) You have two identical equiconvex lenses available; each has diameter d and focal length $f > 0$. The system is to be used in light with wavelength λ_0 .
 - (a) Sketch the afocal system that can be constructed from these two lenses.
 - (b) A zero-power aperture of diameter $\frac{d}{2}$ is placed at the plane halfway between the two lenses in the system sketched in (a). If the object is a point source located at ∞ , determine which element is the stop and use this information to add the marginal and chief rays to the sketch.
 - (c) Now consider a planar object located at a distance $s_1 = f$ “in front” of the first lens and oriented perpendicular to the optical axis. Locate and characterize the image.
 - (d) A small opaque spot is placed in the system exactly at the center of the zero-power aperture between the two lenses. Explain what happens to the image created by the system of the “on-axis” point source located at ∞ from part (b). Describe the effect of varying the diameter of this small spot on the observed output.
 - (e) A second point source that emits the same wavelength λ_0 is added at an infinite distance from the first lens but that is “off axis,” so that the wavefronts from the second source are “tilted” relative to the optical axis. Describe the output created by the system in two cases: before and after the small opaque spot is inserted between the two lenses.

2. (25%) You have been given the job of designing an imaging system using a CCD sensor whose square pixels are $10 \mu\text{m} \times 10 \mu\text{m}$. Assume that there are no “gaps” between the pixels so that all photons that reach the sensor are imaged. The diameter of the optic is d and the focal length is f .
 - (a) Determine the $f/\#$ of the system such that the diameter of the central core of the diffraction spot (the circle enclosed by the first zero of the Fraunhofer diffraction pattern) is fully enclosed in a CCD pixel; assume that $\lambda_0 = 550 \text{ nm}$.
 - (b) Determine the focal length of the system that results in an “angular resolution” (also called a “plate scale”) of 1 arcsecond per pixel at the same wavelength.
 - (c) Use the result of part (b) to determine the diameter of the optic that also satisfies the condition in part (a).

3. (25%) Two point sources that emit the same wavelength λ_0 are located at $[x, y, z] = [\pm \frac{d}{2}, 0, 0]$. The light from these sources is observed at an observation plane parallel to the $x - y$ plane and centered at $[0, 0, z_1]$, where z_1 is sufficiently large that the wavefronts from the sources may be modeled as planes. The waves are added to create an interference pattern.
- Describe and sketch the irradiance pattern at the observation plane. You don't have to "derive" the equation for the pattern, but your description and your graph should be quantitative.
 - The irradiance at the observation plane is recorded on a photographic emulsion so that the transmission $t[x, y]$ of the emulsion is identical to the normalized "complement" of the incident irradiance, i.e., the transmittance is "0" where the irradiance is its maximum and "1" where the irradiance is zero. In other words, the transmittance pattern of the developed film is the "negative" of the irradiance. This recording process is assumed to be exactly linear. Sketch the transmittance $t[x, y]$ function of this "transparency."
 - The developed film with the transmittance pattern $t[x, y]$ is placed in its original location $[0, 0, z_1]$ and illuminated with light from a single on-axis point source at the origin of coordinates $[x, y, z] = [0, 0, 0]$. The light that passes through the transparency then propagates a large distance z_2 into the Fraunhofer diffraction region (so that the new observation plane is located at $[0, 0, z_1 + z_2]$). Sketch and describe the irradiance pattern that is observed at the new observation plane. You may ignore any constant terms; just show the functional form of the measured irradiance.
4. (25%) A digital image $f_q[n, m]$ includes two distinct spatial regions created from random "noise," i.e., unpredictable variations of nonnegative integers. These two regions have the same mean gray values but different variances (i.e., one region is "noisier"); an example is shown. Your mission (should you decide to accept it) is to segment the two regions by applying pixel (point) operators and linear shift-invariant local neighborhood operations (i.e., convolutions). The only nonlinear operations available to you are pixel operations (e.g., thresholding, nonlinear scaling of gray values).
- Sketch the histogram of the original image $f_q[n, m]$.
 - Specify the sequence of operations to segment the regions. (HINT: you need to convert "noisiness" to "gray level" and then select a threshold). Explicitly define any kernels used in convolutions.
 - Sketch approximate histograms that result from each step in your sequence.



Magnified view of section of $f_q[n, m]$ near boundary between two regions of noise with the same mean and different variances.

5. (15%) Consider the statement: “In normal use, the magnitude of the transverse magnification of an imaging system increases as its equivalent focal length is increased.” Find an expression for the transverse magnification in terms of the focal length that demonstrates the truth or falsehood of this statement. State any conditions that must be satisfied in the mathematical expression (e.g., why does the sentence include the caveat “in normal use”?).
6. (15%) Histogram equalization:
- A discrete image $f_q[n, m]$, where f_q is an integer in the range $0 \leq f_q \leq 2^m - 1$ is processed to create the image $g_q[n, m]$ with a “flat” histogram; $g_q[n, m]$ is then subjected to the same “flattening” operation to produce $p_q[n, m]$. Describe the similarities and differences among the three images.
 - Comment on the value and the problems with the use of histogram equalization to compare images of the same scene taken under different conditions.
7. (15%) Information Content:
- Derive the Huffman code for a 3-bit image with a flat histogram. (SHOW YOUR WORK!)
 - The gray values of an image are f_q where $0 \leq f_q \leq 2$ and the probability of each level ($p[f_q] \leq 1$) is not know. How many unique Huffman codes exist for this image?
8. (10%) Classify the action of the following kernels (e.g., “highpass,” “lowpass”) and specify an application for each (all locations not shown have value “0”).

(a)

0	0	0
-1	+1	0
0	0	0

(b)

0	0	$-\frac{1}{2}$
0	+2	0
$-\frac{1}{2}$	0	0

(c)

+1	0	-1	0	+1	0	-1	0	+1	0	-1
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(d)

+2	-1	-1
-1	+2	-1
-1	-1	+2

9. (10%) The gradient operator $\nabla f [n, m]$ produces a vector at each pixel. The magnitude of the gradient is defined to be the square root of the sum of the squares of the vector components, and is sometimes approximated as the sum of the absolute values of the vector components:

$$\begin{aligned} |\nabla f [n, m]| &= \sqrt{[(\nabla f [n, m])_x]^2 + [(\nabla f [n, m])_y]^2} \\ &\cong |(\nabla f [n, m])_x| + |(\nabla f [n, m])_y| \end{aligned}$$

The azimuth angle of the gradient is defined as:

$$\Phi \{ \nabla f [n, m] \} = \tan^{-1} \left[\frac{(\nabla f [n, m])_y}{(\nabla f [n, m])_x} \right]$$

- (a) Evaluate these two statements for the magnitude of the gradient of a binary (or *bitonal*) image $f [n, m]$ of size $N \times N$ pixels that contains a rectangular region of level “1” surrounded by pixels with level “0.”
- (b) What are the possible edge “directions” that would be evaluated for this image?

10. **Optional Extra Credit Bonus Question:**

The “Quote for the day” is the first line of the poem, “In the Apartments of the Divorced Men” by Sue Ellen Thompson, from **The Leaving: New and Selected Poems**:

The apartments of the divorced men are small, you can stand in the doorway and see their whole lives as through a convex lens, the way a fish sees all the ocean.

Comment on the optical system described in this sentence on technical and figurative levels. You can receive up to 10 real-valued points for your technical description and up to 10 imaginary-valued points for your figurative discussion. Your extra-credit score will be the magnitude of your complex-valued score.

Possibly useful information:

$$\begin{aligned} \cos \theta &= \sum_{n=0}^{\infty} (-1)^n \frac{\theta^{2n}}{(2n)!} \\ \sin \theta &= \sum_{n=0}^{\infty} (-1)^n \frac{\theta^{2n+1}}{(2n+1)!} \\ \frac{1}{1-t} &= \sum_{n=0}^{\infty} t^n \text{ for } |t| < 1 \end{aligned}$$

$$\begin{aligned} \varphi &= \frac{1}{f} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \\ F[k] &\propto \sum_{n=0}^{N-1} f[n] \exp \left[-2\pi i \frac{nk}{N} \right] \end{aligned}$$