

SIMG-712-01-20042 Homework #2
Due W, 12/15/2004

1. Show that the following waveforms are solutions to the differential wave equation

$$\begin{aligned}\psi_a [z, t] &= A_0 \sin [k_0 (z - v_0 t)] \\ \psi_b [z, t] &= A_0 \cos [k_0 z - \omega_0 t]\end{aligned}$$

2. A waveform has the shape:

$$y [z, t = 0] = \frac{C}{1 + (2\pi z)^2}$$

where C is some numerical constant.

- (a) Sketch the profile of the wave $y [t, 0]$.
 - (b) Write an expression for the wave as a function of z and t if it travels with velocity v_0 towards $z = -\infty$.
 - (c) Assume that $v = 1 \frac{\text{m}}{\text{s}}$. Sketch the profile $y [z, t = 2 \text{ s}]$
3. An isotropic monochromatic point source radiates at angular temporal frequency of ω_0 in vacuum with a power of 100 W.
- (a) What is the flux density at a distance of 1 m?
 - (b) What are the amplitudes of the **E**- and **B**-fields at that distance?
 - (c) Repeat for a distance of 2 m.
4. Consider cylindrical waves emitted by a line source

- (a) Make an argument from conservation of energy considerations that cylindrical waves must have an amplitude that decreases approximately as $\rho^{-\frac{1}{2}}$, where ρ is the radial coordinate in a cylindrical coordinate system and the waves are originating from a line at $\rho = 0$.
- (b) Show explicitly that:

$$\psi(\rho, t) = \psi_0 \frac{\exp i(k_0 \rho - \omega_0 t)}{\sqrt{\rho}}$$

is a solution to the three-dimensional wave equation,

$$\nabla^2 \psi - \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} = 0$$

in cylindrical coordinates for large ρ .

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5. Our simple model of dispersion assumed that the response of an electron to a disturbance from its equilibrium position is a decaying oscillation, which we called the impulse response of the electron, e.g.,

$$h [t] = A_0 \exp [-\gamma_0 t] \cdot STEP [t] \cdot \sin [2\pi\nu_0 t]$$

From this we derived the electron amplitude as a function of the frequency ν of the incident light; we called it $H [\nu]$, and we outlined how this leads to the index of refraction. Here, assume that the impulse response of the electron motion to a disturbance includes two different sinusoidal frequencies that decay from different amplitudes at different rates. Evaluate and plot the frequency response of this system as real and imaginary parts, and as magnitude and phase.