

SIMG-712-20042 Homework #1
Due W, 12/8/2004

NOTE: These are “review” problems intended to encourage reading of the review material and setting the stage for more difficult problems to come...

1. A source of harmonic motion of the form $y(t) = 6 \cdot \cos(\omega_0 t)$ located at the origin of the spatial coordinate system emits a wave that travels through a uniform (i.e., homogeneous) medium at a rate of 60 mm per second.
 - (a) Find the formula for the displacement due to this wave at a distance of 800 mm from the origin.
 - (b) Find the displacement at that distance for $t = 60$ s.
(HINT: It is ALWAYS useful to draw a diagram of the problem before trying to solve it!)

2. What is the phase difference (in radians) between any two points on a harmonic electromagnetic wave separated by $\Delta z = 1 \mu\text{m}$ (1 micron) if the wavelength is $\lambda = 550 \text{ nm}$?
3. $N > 1$ simple temporal harmonic oscillatory motions with the same amplitude and temporal frequency are superimposed (summed), i.e., the output $g[t]$ may be written as:

$$g[t] = \sum_{n=1}^N A_0 \cos[2\pi\nu_0 t + \phi_n]$$

The phase difference between successive pairs of oscillations is:

$$\Delta\phi_n = \phi_n - \phi_{n-1}, \text{ where } n = 2, 3, \dots, N$$

If the phase difference between each successive pair is invariant, find one value of this phase difference as a function of N for which the amplitude of the sum is zero. Again, it may be useful to draw a picture to help you solve the problem, and it also may be useful to consider the result for small integer values of N .

4. Use the Euler relation: $e^{i\theta} = \cos[\theta] + i \sin[\theta]$ to derive expressions for the following in terms of $\cos[\theta_1]$, $\sin[\theta_1]$, $\cos[\theta_2]$, and $\sin[\theta_2]$:
 - (a) $\sin[2\theta_1]$
 - (b) $\cos[\theta_1 \pm \theta_2]$
 - (c) $\sin[\theta_1 \pm \theta_2]$
5. Use the formulas for $\cos[\theta_1 \pm \theta_2]$ and for $\sin[\theta_1 \pm \theta_2]$ just derived to derive expressions for the following in terms of the sum and difference frequencies $\omega_1 \pm \omega_2$. ALSO plot the results for $\omega_1 = 1$ and $\omega_2 = \frac{3}{2}$ radian per second, respectively.
 - (a) $\sin[\omega_1 t] \cdot \sin[\omega_2 t]$

- (b) $\cos [\omega_1 t] \cdot \cos [\omega_2 t]$
- (c) $\sin [\omega_1 t] \pm \sin [\omega_2 t]$
- (d) $\cos [\omega_1 t] \pm \cos [\omega_2 t]$

6. Consider the superposition of two sinusoidal traveling waves:

$$\begin{aligned}
 f_1 [z, t] &= A_1 \cos [k_1 z - \omega_1 t], \\
 A_1 &= 10 \text{ mm}, \nu_1 = 1000 \text{ Hz}, v_1 = 250 \frac{\text{m}}{\text{s}} \\
 f_2 [z, t] &= A_2 \cos [k_2 z - \omega_2 t], \\
 A_2 &= 9 \text{ mm}, \nu_2 = 1500 \text{ Hz}, v_2 = 500 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

- (a) Find an expression for the resulting wave in terms of the average wave, the modulation wave, plus any remaining amplitude.
 - (b) Calculate the wavelengths of the average and modulation waves.
 - (c) Find the velocities of the average and modulation waves.
 - (d) Does this system exhibit normal or anomalous dispersion?
7. The phase velocity of waves in some medium is proportional to $\omega^{+\frac{1}{4}}$. Find an expression for the modulation velocity and determine whether the waves exhibit normal or anomalous dispersion.