

CLASS PROBLEM:

A. An image measurement requires that you know the effective measurement aperture area at the sample film plane. You estimate the magnification of the optics, M . You know that you can calculate the detected image region by relating each sensor aperture dimension

$$x_{image} = \frac{1}{M} x_{sensor}$$

where x_{image} and x_{sensor} are corresponding linear dimensions in mm. The estimated value of M (and therefore $1/M$), however, contains an error and can be considered a random variable. Let $Q=1/M$, with a 5% relative error,

$$\epsilon_Q = \frac{\sigma_Q}{\mu_Q} = 0.05$$

If you know the actual sensor aperture is 2mm x 200 μm . and the mean magnification is 30 ($\mu_Q = 1/30$), what is the error in the estimated measuring aperture on the film, ie. the mean value and standard deviation of the image aperture area ?

$$x_i = \frac{1}{m} x_s$$

or

$$A_i = \left(\frac{1}{m}\right)^2 A_s \quad (1)$$

$$A_s = (2\text{mm} \times 0.2\text{mm}) = 0.4\text{mm}^2$$

Let $Q = \frac{1}{m}$ a random variable

$$\mu_Q = 0.033, \quad \epsilon_Q = \frac{\sigma_Q}{\mu_Q} = 0.05$$

so

$$\sigma_Q = (0.05)(0.033) = 1.67 \times 10^{-3}$$

1. MEAN

From (1)

$$A_i = Q^2 A_s$$

$$\mu_{A_i} = E[Q^2 A_s] = A_s E[Q^2] \quad (2)$$

$$E[Q^2] = \cancel{\mu_Q^2} + \sigma_Q^2 + \mu_Q^2$$

$$= (1.67 \times 10^{-3})^2 + (0.033)^2$$

$$= 2.78 \times 10^{-6} + 1.11 \times 10^{-3}$$

↑ v. small

$$= 1.11 \times 10^{-3} \quad (3)$$

Substitute (3) into (2)

$$\begin{aligned} \mu_{A_i} &= A_s \mu_Q^2 \\ &= (0.4\text{mm}^2)(1.11 \times 10^{-3}) \\ &= 4.45 \times 10^{-4} \text{mm}^2 \end{aligned}$$

2. VARIANCE

$$A_i = Q^2 A_s$$

$$\sigma_{A_i}^2 = \left(\left. \frac{dA_i}{dQ} \right|_{\mu_Q} \right)^2 \sigma_Q^2 \quad (1)$$

$$\frac{dA_i}{dQ} = 2QA_s$$

$$\left. \frac{dA_i}{dQ} \right|_{\mu_Q} = 2A_s / \mu_Q \quad (2)$$

Substitute (2) into (1)

$$\sigma_{A_i}^2 = 4A_s^2 / \mu_Q^2 \sigma_Q^2$$

$$\sigma_{A_i} = 2A_s / \mu_Q \sigma_Q$$

$$= (2)(0.4)(0.033)(1.67 \times 10^{-3})$$

$$= 4.45 \times 10^{-5} \text{ mm}^2$$

$$= 44.5 \mu\text{m}^2$$