

TURN OFF AND PUT AWAY CELL PHONES

UNPROGRAMMED CALCULATORS OKAY

Select 3 of the 5 problems ($\frac{1}{3}$ each). Staple problems together IN NUMERICAL ORDER. Show your work. If you solve problems “by inspection”, then write down the thought process that led to your conclusion.

YOU MAY SUBMIT **ONE** ADDITIONAL PROBLEM FOR EXTRA CREDIT

SPECIFY WHICH AND STAPLE BEHIND THE THREE PROBLEMS

1. We used the Argand diagram (also called the phasor diagram) to represent temporal oscillatory motion.
 - (a) Use the Argand diagram to demonstrate that the superposition of three oscillations with identical frequencies is an oscillation with that frequency.
 - (b) Use the Argand diagram to find the sum of these three oscillations:

$$\begin{aligned} f_1 [t] &= \cos \left[2\pi\nu_0 t + \frac{\pi}{3} \right] \\ f_2 [t] &= \cos \left[2\pi\nu_0 t - \frac{\pi}{3} \right] \\ f_3 [t] &= -\cos [2\pi\nu_0 t] \end{aligned}$$

where ν_0 is arbitrary.

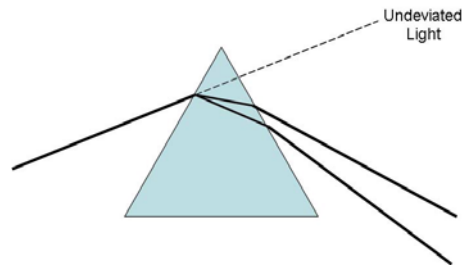
2. A function $f_1 [z, t]$ is created by summing two sinusoidal functions

$$f_1 [z, t] = 2 \cos \left[-2\pi \frac{z}{\lambda_0} - 2\pi\nu_0 t - \frac{\pi}{2} \right] - 2 \cos \left[2\pi \frac{z}{\lambda_0} - 2\pi\nu_0 t - \frac{\pi}{2} \right]$$

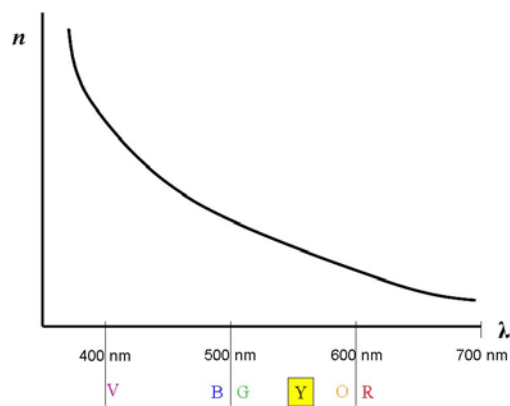
Find an expression for the result as the *product* of two sinusoidal functions.

3. Consider four ideal linear polarizers oriented at $\theta_1 = 0^\circ$, $\theta_2 = 30^\circ$, $\theta_3 = 60^\circ$, and $\theta_4 = 90^\circ$. Natural light with “intensity” I_0 (proportional to E_0^2) is incident on the first polarizer such that the emerging irradiance is I_1 .
 - (a) Specify the state of polarization of the light emerging from the last linear polarizer.
 - (b) Find the “intensity” or “irradiance” emerging from the last polarizer in terms of I_0 ; you may use any method, but extra credit will be given for use of Jones vectors and matrices.
4. From observations of snowflakes, you might surmise that frozen water has a crystalline structure in a hexagonal form (due to the dominant angular position of the hydrogen atoms at 120°). This anisotropic structure produces different forces on the charges in the molecules, which results in slightly different refractive indices along orthogonal directions. If the indices of refraction along orthogonal directions happen to be $n_1 = 1.313$ and $n_2 = 1.309$, determine the thickness d of ice that would produce a relative phase delay of $\Delta\phi = \frac{\pi}{2}$ for the polarizations along the two directions for light with vacuum wavelength $\lambda_0 = 500$ nm. MORE→→→

5. Light is dispersed by a glass prism to create the spectrum as shown:



- (a) Replicate this sketch on your paper and label the order of the colors that emerge from the prism if the glass exhibits normal dispersion with a curve of this shape (vertical axis of $n[\lambda]$ is arbitrary, but you may assume a reasonable range of values of n)



- (b) Now consider some theoretical substance that exhibits anomalous dispersion with a resonance centered at $\lambda = 500 \text{ nm}$. Use the “undamped” model of the refractive index to sketch a possible graph of $n[\lambda]$ for this material.
- (c) Make another sketch of a prism and label the sequence of dispersed colors that would be produced if the prism were made of the material in part (b). Explain any features on your graph.