1 Laboratory #4: "Division-of-Wavefront" Interference

1.1 Theory

Recent labs on optical imaging systems have used the concept of light as a "ray" in geometrical optics to model the action of lenses. We were able to determine the locations of images and their magnifications. However, the concept of light as a "wave" also is fundamental to imaging, particularly in its manifestation in "diffraction", which is the fundamental limitation on the action of an optical imaging system. "Interference" and "diffraction" may be interpreted as the same phenomenon, differing only in the number of sources involved (interference $\Longrightarrow$ few sources, say 2 - 10, whereas diffraction $\Longrightarrow$ many sources, up to an infinite number). In this lab, the two sources are obtained by dividing the wave emitted by a single source by introducing two apertures into the system to divide the wavefront into two sections that are recombined to form interference; this is "division-of-wavefront" interference. In the next lab, we will divide the light by introducing a beamsplitter to create "division-of-amplitude interferometry".

1.1.1 Interference:

We introduce interference by recalling the expression for the sum of two sinusoidal temporal oscillations of the same amplitude and different frequencies:

$$\begin{aligned} A_0 \cos[\omega_1 t] + A_0 \cos[\omega_2 t] &= 2A \cos\left[\frac{(\omega_1+\omega_2)}{2} t\right] \cos\left[\frac{(\omega_1-\omega_2)}{2} t\right] \\ &= 2A_0 \cos[\omega_{\text{mod}} t] \ \cos[\omega_{avg} t] \end{aligned}$$

and the travelling-wave analogue:for two plane waves propagating along the z axis:

$$A_0 \cos[k_1 z - \omega_1 t] + A_0 \cos[k_2 z - \omega_2 t] = 2A_0 \cos[k_{\text{mod}} z - \omega_{\text{mod}} t] \cos[k_{avg} z - \omega_{avg} t]$$

$$k_{\text{mod}} = \frac{k_1 - k_2}{2},\ \omega_{\text{mod}} = \frac{\omega_1 - \omega_2}{2},\ k_{avg} = \frac{k_1 + k_2}{2},\ \omega_{avg} = \frac{\omega_1 + \omega_2}{2}$$

If the medium exhibits normal dispersion or is not dispersive, the resulting wave is the product of a (slow) traveling wave with velocity $v_{\text{mod}} = \frac{\omega_{\text{mod}}}{k_{\text{mod}}}$ and a (faster) traveling wave with velocity $v_{avg} = \frac{\omega_{avg}}{k_{avg}}$. Thus far, we have described traveling waves directed along one axis (usually z), but the equations may be generalized easily to model waves traveling in any direction. Instead of a scalar angular wavenumber k, we define the 3-D *wavevector*:

$$\underline{\mathbf{k}} = [k_x, k_y, k_z] = \hat{\underline{\mathbf{x}}} k_x + \hat{\underline{\mathbf{y}}} k_y + \hat{\underline{\mathbf{z}}} k_z$$

which points in the direction of travel of the wave. The length of the wavevector is proportional to λ_0^{-1}.

$$|\underline{\mathbf{k}}| = \sqrt{k_x^2 + k_y^2 + k_z^2} = \frac{2\pi}{\lambda_0}.$$

Thus the equation for a traveling wave in 3-D space with angular temporal frequency ω_0 becomes:

$$f[x,y,z,t] = f[\underline{\mathbf{r}}, t] = A_0 \cos[k_x x + k_y y + k_z z - \omega_0 t] = A_0 \cos[\underline{\mathbf{k}} \bullet \underline{\mathbf{r}} - \omega_0 t]$$

For simplicity, we will limit ourselves to the still-useful 2-D case by setting $k_y = 0$:

$$f[x,z,t] = A_0 \cos[k_x x + 0 \cdot y + k_z z - \omega_0 t] = A_0 \cos[k_x x + k_z z - \omega_0 t] = A_0 \cos[\underline{\mathbf{k}} \bullet \underline{\mathbf{r}} - \omega_0 t]$$

We already know that the temporal modulation frequency of the sum of two 1-D waves with angular frequency ω_0 is $\frac{\omega_0 - \omega_0}{2} = 0$, which produces standing waves. The 2-D or 3-D case is different;

the wavefront can exhibit a periodic variation in the phase $\phi[\underline{\mathbf{r}}, t] = \underline{\mathbf{k}} \bullet \underline{\mathbf{r}} - \omega t$, even if $\omega_1 = \omega_2 = \omega \rightarrow \lambda_1 = \lambda_2 = \lambda_0$. If light from a single source is divided into two sections by introducing two apertures into the system, Huygens' principle indicates that the light through the two apertures will "spread" and recombine. When viewed at a single location, the two "beams" of light with the same wavelength will recombine with different wavevectors such that $|\underline{\mathbf{k_1}}| = |\underline{\mathbf{k_2}}| = |\underline{\mathbf{k}}|$. This happens when the Cartesian sum of the components of $\underline{\mathbf{k}}$ are the same, but

$$[(k_x)_1, (k_y)_1] \neq [(k_x)_2, (k_y)_2]$$

Note that the following simple result is true only for

$$\lambda_1 = \lambda_2.$$

Light of the same wavelength (and same optical frequency) is described as *coherent*. In this case, consider the superposition of two plane waves of the same optical frequency ω, one traveling in direction $\underline{\mathbf{k}}_1$ and one in direction $\underline{\mathbf{k}}_2$:

$$\begin{aligned} f_1[x, y, z, t] &= A_0 \cos[\underline{\mathbf{k}}_1 \bullet \underline{\mathbf{r}} - \omega_0 t] \\ f_2[x, y, z, t] &= A_0 \cos[\underline{\mathbf{k}}_2 \bullet \underline{\mathbf{r}} - \omega_0 t], \end{aligned}$$

where $\underline{\mathbf{k}}_1 = [k_x, 0, k_z]$ and $\underline{\mathbf{k}}_2 = [-k_y, 0, k_z]$, *i.e.*, the wavevectors differ only in the sign of the y-component. The wavevectors have the same length:

$$|\underline{\mathbf{k}}_1| = |\underline{\mathbf{k}}_2| = \frac{2\pi}{\lambda_0} \Longrightarrow \lambda_1 = \lambda_2 \equiv \lambda_0$$

The x and z components of the wavevectors are:

$$\begin{aligned} k_z &= |\underline{\mathbf{k}}| \cos[\theta] = \frac{2\pi}{\lambda_0} \cos[\theta] \\ k_x &= \frac{2\pi}{\lambda_0} \sin[\theta]. \end{aligned}$$

The superposition of the electric fields is:

$$f_1[x, y, z, t] + f_2[x, y, z, t] = A_0 \{\cos[(k_z z - \omega_0 t) + k_x x] + \cos[(k_z z - \omega_0 t) - k_x x]\}$$

which can be recast using the formula for $\cos[\alpha \pm \beta]$ to:

$$\begin{aligned} f_1[x, y, z, t] + f_2[x, y, z, t] &= 2A_0 \cos[k_x x] \cos[k_z - \omega_0 t] \\ &= 2A_0 \cos\left[\left(\frac{2\pi}{\lambda_0} \sin[\theta]\right) x\right] \cdot \cos[k_z z - \omega_0 t] \end{aligned}$$

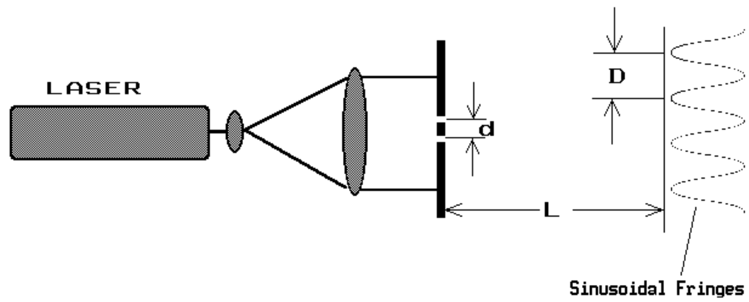
Note that there is no time dependence in the first term: this is a time-invariant pattern. Also recall that the measured quantity is the *intensity* (also called the *irradiance*) of the pattern, which is the time average of the squared magnitude. The time-varying term travels at the velocity of light and thus its rapid oscillation is not visible. The visible pattern consists of only the time-invariant term:

$$\begin{aligned} |f_1[x, y, z, t] + f_2[x, y, z, t]|^2 &\propto 4A_0^2 \cos^2\left[\frac{2\pi x}{\lambda_0} \sin[\theta]\right] \\ &= 4A_0^2 \cdot \frac{1}{2} \left(1 + \cos\left[\frac{4\pi x}{\lambda_0} \sin[\theta]\right]\right) \\ &\propto 2A_0^2 \cos\left[\frac{2\pi x}{D}\right], \textit{ where } D \equiv \frac{\lambda_0}{2 \cdot \sin[\theta]} \end{aligned}$$

where the identity $\cos^2[\beta] = \frac{1}{2}(1 + \cos[2\beta])$ has been used. The intensity pattern has a cosine form (maximum at the center) and a period proportional to λ_0 and inversely proportional to $\sin[\theta]$. If θ is small, the period D of the pattern is long. If the distance L to the observation plane is large, then $\sin[\theta] \cong \frac{d}{L}$ and the period of the pattern is approximately:

$$D \cong \frac{\lambda_0 L}{d} \rightarrow \boxed{Dd \simeq L\lambda_0}$$

1.2 Equipment



Interference pattern formed from two very small holes is an (approximately) sinusoidal pattern with period $D = \frac{L\lambda_0}{d}$

1. He:Ne Laser;
2. beam expanding lens (ideally, a "spatial filter" consisting of a microscope objective and a pinhole aperture, but a short-focal length positive lens will work);
3. positive lens after the beam expander to "recollimate" the light (i.e., "focus" the light at $z = +\infty$);
4. "perfboard" (a piece of fiberboard with a regular grid of small holes used as breadboards for electronic circuits);
5. black tape (to block holes in the perfboard);
6. aluminum foil, single-edged razor blades, and needles, to make objects for diffraction;
7. Slide #1 from the set of Metrologic slides (be careful to return all of them!)

1.3 Procedure:

1. Use a piece of perfboard as the object. Perfboard is used to wire electronic circuits and has holes of approximately equal size spaced at equal intervals (often $\frac{1}{10}$ in $\cong 2.5\,\text{mm}$). Use the black tape to block light from all holes in the perfboard save one. Measure the "image" (distribution of light) at an observation plane located at a "large" distance L from the object ($L \gtrapprox 1\,\text{m}$); you may record the pattern either photographically or by sketch. Include measurements or estimates of the dimensions and spacings of the features in the pattern.
2. Open up one of the adjacent holes to create an object with two holes separated by $d = \frac{1}{10}$ inch; record the "image" at the output plane
3. Record "images" created by two holes separated by a larger interval $d_2 = 0.2$ inch;
4. Record "images" created by many holes.
5. Use the results of the previous steps and measurements of L, D, and d to determine the approximate wavelength λ_0 of the He:Ne laser (actual $\lambda_0 = 632.8\,\text{nm}$).

6. Repeat the same procedure using an object created by piercing aluminum foil with a needle. It is easy to make holes that are quite "round" by rotating the needle between your fingers as you pierce the foil. Use your finger or a piece of cardboard as an "anvil" when you make the hole. It also is easy to vary the sizes of the holes by varying the pressure of the needle on the anvil. Record differences between "images" created by a single very tiny hole and by a single larger hole.

7. Make an object consisting of two holes of approximately equal diameter. Use the measured interval D of the fringe pattern, the distance L, and the known wavelength λ_0 to calculate the separation d between the two holes.

8. Repeat using slide #1 from the set of Metrologic slides; this is a set of 8 small holes on a dark background.

9. Make objects consisting of a single "large" aperture and two "large" apertures ($d \cong 5-10\,\text{mm}$) and view the pattern at both "large" and "small" distances (e.g., $L \cong 1\,\text{m}$ and $L \cong 200\,\text{mm}$). Record the observed pattern. The pattern for a single large aperture observed at a short distance away is an example of a *Fresnel diffraction pattern*.

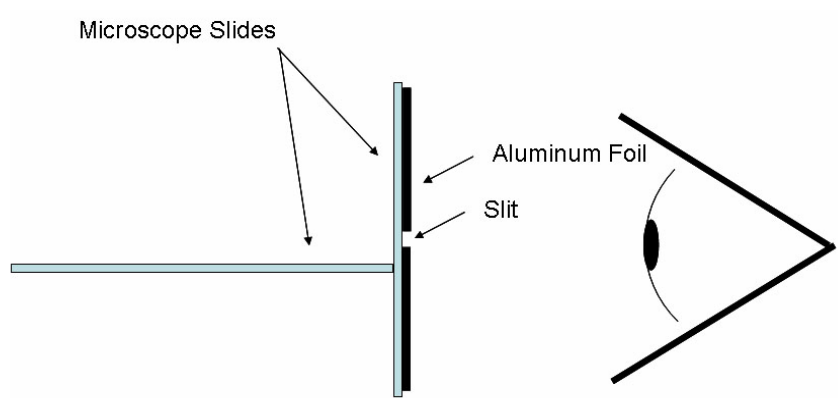
10. Make objects from slits in the aluminum foil (use the razor blade); try one, two, and several slits at approximately the same spacing?

11. (Optional) Lloyd's Mirror

    (a) Make a single slit and a double slit in this way: tape the edges of a piece of aluminum foil to a microscope slide. Cut the slits with a razor blade so that they are separated by 0.5 mm or less. Make one slit longer than the other (say, 5 mm or so) so that you can switch from one slit to two easily. Hold the single slit close to one eye and look at a white light source. Estimate the angular width of the central maximum by making marks on a piece of paper that is behind the light source to give the scale (measure the distances to get the angular size). Then measure the slit width using a magnifying glass or optical comparator.

    (b) Look at a white light source (e.g., the sky or a frosted light bulb) through the double slit. Describe what you see.

    (c) Now make a Lloyd's mirror by taking a second microscope slide and holding it as shown in the figure (you might also try the same configuration except with the aluminum foil slit on the opposite side from your eye. You can stick the slides together with duct tape modeling clay, or putty. Adjust the "mirror" to get as narrow a separation between the slit and its "image" as possible, say 0.5 mm. Bring the assembly to one eye and focus on a source at a "large" distance away. Look for a few (3 or 4) black "streaks" parallel to the slit; these are the "zeros" of the interference pattern due to destructive interference of the light directly from the source and reflected from the horizontal microscope slide. The light from these two are always coherent (regardless of the source!) but out of phase by 180° due to the phase change on reflection.



2 Questions

1. Characterize the differences in the patterns produced by one, two, and many "small" round apertures in the perfboard (so that $L >> d$) ; in particular, can you see if some part(s) of the pattern is the same regardless of the number of apertures?

2. Characterize the differences in the patterns produced by two "small" apertures ($L >> d$) and by two "large" apertures ($L > d$).