1051-455-20073, Physical Optics

1 Laboratory #3 – POLARIZED LIGHT

1.1 Background:

This lab introduces the concept of *polarization* of light. As we have said in class (and as is obvious from the name), electromagnetic radiation requires two traveling-wave vector components to propagate: the electric field, often specified by $\underline{\mathbf{E}}$, and the magnetic field $\underline{\mathbf{B}}$. The two components are orthogonal (perpendicular) to each other, and mutually orthogonal to the direction of travel, which is often specifed by the vector quantity $\underline{\mathbf{s}}$ (the Poynting vector):

$$\underline{\mathbf{s}} \equiv \underline{\mathbf{E}} \times \underline{\mathbf{B}}$$

where "×" specifies the mathematical "cross product".

1.1.1 Cross Product

Just to review, the cross product of two arbitrary three-dimensional vectors $\underline{\mathbf{a}} = [a_x, a_y, a_z]$ and $\underline{\mathbf{b}} = [b_x, b_y, b_z]$ is defined:

$$\begin{aligned} \underline{\mathbf{a}} \times \underline{\mathbf{b}} &\equiv \det \begin{bmatrix} \hat{\underline{\mathbf{x}}} & \hat{\underline{\mathbf{y}}} & \hat{\underline{\mathbf{z}}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{bmatrix} \\ &\equiv \hat{\underline{\mathbf{x}}} (a_y b_z - b_y a_z) + \hat{\underline{\mathbf{y}}} (a_z b_x - b_z a_x) + \hat{\underline{\mathbf{z}}} (a_x b_y - a_y b_x) \end{aligned}$$

where $\hat{\underline{\mathbf{x}}}$, $\hat{\underline{\mathbf{y}}}$, and $\hat{\underline{\mathbf{z}}}$ are the unit vectors directed along the respective Cartesian axes and $\det[\,]$ represents the evaluation of the *determinant* of the 3×3 matrix. You may recall that the "cross product" is defined ONLY for 3-D spatial vectors.

For example, if the electric field $\underline{\mathbf{E}}$ is oriented along in the $\boldsymbol{x}$-direction with amplitude E_x (so that $\underline{\mathbf{E}} || \hat{\underline{\mathbf{x}}}$, where $||$ indicates "is parallel to"). The electric field is $\underline{\mathbf{E}} = \hat{\underline{\mathbf{x}}} \cdot E_x + \hat{\underline{\mathbf{y}}} \cdot 0 + \hat{\underline{\mathbf{z}}} \cdot 0$. Consider also that the magnetic field $\underline{\mathbf{B}} \perp \underline{\mathbf{E}}$ is oriented along the $\boldsymbol{y}$-direction ($\underline{\mathbf{B}} || \hat{\underline{\mathbf{y}}}$, $\underline{\mathbf{B}} = \hat{\underline{\mathbf{x}}} \cdot 0 + \hat{\underline{\mathbf{y}}} \cdot B_y + \hat{\underline{\mathbf{z}}} \cdot 0$), then the electromagnetic field travels in the direction specified by:

$$\begin{aligned} \hat{\underline{\mathbf{E}}} \times \hat{\underline{\mathbf{B}}} &\equiv \det \begin{bmatrix} \hat{\underline{\mathbf{x}}} & \hat{\underline{\mathbf{y}}} & \hat{\underline{\mathbf{z}}} \\ E_x & 0 & 0 \\ 0 & B_y & 0 \end{bmatrix} \\ &= \hat{\underline{\mathbf{x}}} (0 \cdot 0 - B_y \cdot 0) + \hat{\underline{\mathbf{y}}} (E_x \cdot 0 - 0 \cdot 0) + \hat{\underline{\mathbf{z}}} (E_x \cdot B_y - 0 \cdot 0) \\ &= \hat{\underline{\mathbf{z}}} (E_x \cdot B_y) \end{aligned}$$

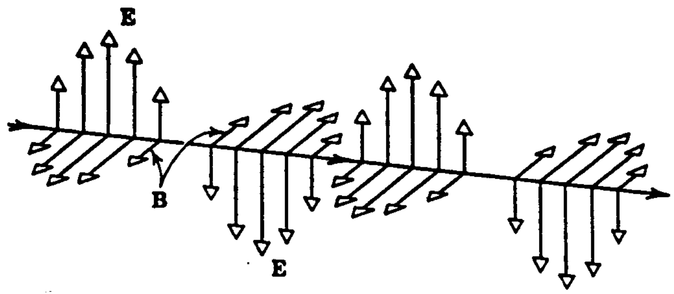
Thus the electromagnetic wave propagates in the direction of the positive z axis.

1.1.2 Polarization

In vacuum (sometimes called *free space*), the electric and magnetic fields propagate in phase, which means that both have extrema or nulls at the same locations in space-time. For example, if the electric field is a traveling wave with phase $k_0 z - \omega_0 t + \phi_0$, then the two component fields are:

$$\begin{aligned} \underline{\mathbf{E}}[z,t] &= \hat{\underline{\mathbf{x}}} E_0 \cos[k_0 z - \omega_0 t + \phi_0] \\ \underline{\mathbf{B}}[z,t] &= \hat{\underline{\mathbf{y}}} B_0 \cos[k_0 z - \omega_0 t + \phi_0] = \hat{\underline{\mathbf{y}}} \left(\frac{E_0}{c}\right) \cos[k_0 z - \omega_0 t + \phi_0] \end{aligned}$$

where c is the velocity of light in vacuum. Of course, we know the relationships of the wavenumber $k_0 = 2\pi/\lambda_0$, the temporal angular frequency $\omega_0 = 2\pi\nu_0 = 2\pi c/\lambda_0$, and the veloctiy $c = \lambda_0 \nu_0$. Because the amplitude of the electric field is larger by a factor of c, the phenomena associated with the electric field are easier to measure. Most of the force exerted by the electromagnetic field on a charge comes form the electric field $\underline{\mathbf{E}}$ (which may vary with time/position), so it is the direction of $\underline{\mathbf{E}}$ that is called the *polarization*.



For waves in vacuum, the electric and magnetic fields are "in phase" and the wave travels in the direction specified by $\hat{\mathbf{s}} = \hat{\mathbf{E}} \times \hat{\mathbf{B}}$.

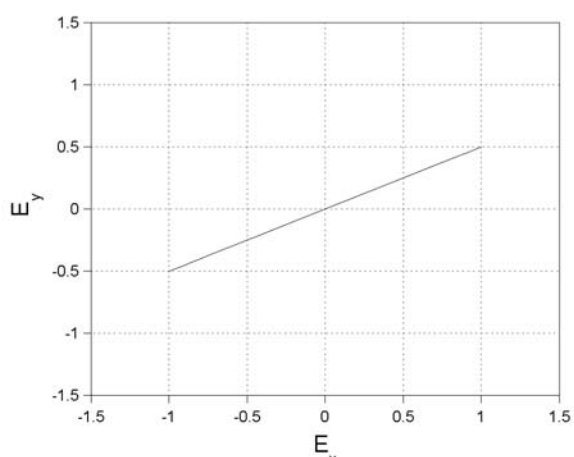
As a side comment, note the difference between the *polarization* of the E-M wave and the *polarizability* of a material, which is a measure of the effect of the electric field on the bound charges in the material.

Polarized light comes in different flavors: plane (or linear), elliptical, and circular. In the first lab on oscillations, we introduced some of the types of polarized light when we added oscillations in the x- and y-directions (then called the *real* and *imaginary parts*) with different amplitudes and initial phases. The most commonly referenced type of light is plane polarized, where the the electric field vector $\underline{\mathbf{E}}$ points in the same direction for different points in the wave. Plane-polarized waves with $\underline{\mathbf{E}}$ oriented along the x- or y-axis are easy to visualize; just construct a traveling wave oriented along that direction. Plane-polarized waves at an arbitrary angle θ may be constructed by adding x- and y-components with the same frequency and phase and different amplitudes, *e.g.,*

$$\underline{\mathbf{E}}[z,t] = \hat{\underline{\mathbf{x}}} E_x \cos[k_0 z - \omega_0 t + \phi_0] + \hat{\underline{\mathbf{y}}} E_y \cos[k_0 z - \omega_0 t + \phi_0]$$

When the two component electric fields are "in phase" (so that the arguments of the two cosines are equal for all $[z,t]$), then the angle of polarization, specified by θ, is obtained by a formula analogous to that for the phase of a complex number:

$$\theta = \tan^{-1}\left[\frac{E_y \cos[k_0 z - \omega_0 t + \phi_0]}{E_x \cos[k_0 z - \omega_0 t + \phi_0]}\right] = \tan^{-1}\left[\frac{E_y}{E_x}\right]$$



Angle of linear polarization for $(E_x)_{\max} = 1$ *and* $(E_y)_{\max} = 0.5$ *is* $\tan^{=1}\left[\frac{E_y}{E_x}\right] \simeq 0.463$ *radians* $\simeq 26.6^\circ$.

Note that the electric fields directed along the x- and y-directions are independent; they need not have the same amplitude, frequency, or initial phase. In this discussion of polarization, the components must have the same frequency, but may have different amplitudes (as just shown) and/or initial phases. If the amplitudes are equal to E_0 but the initial phases differ by $\pi/2$ radians, we have:

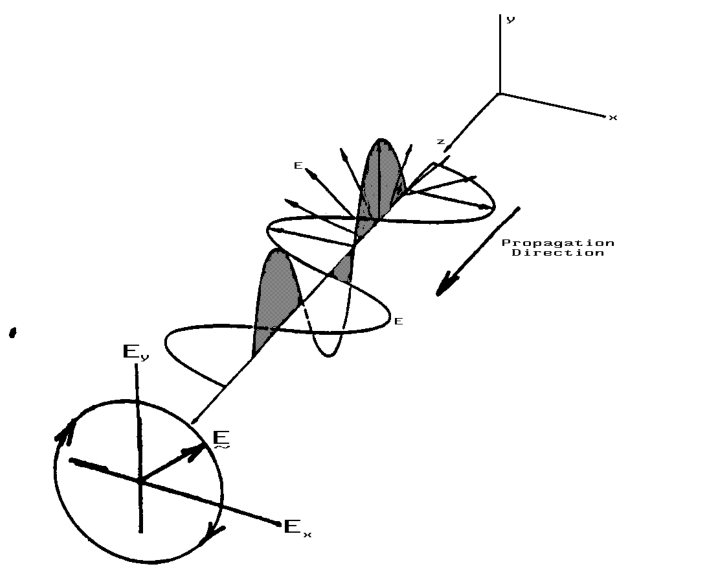
$$\underline{\mathbf{E}}[z,t] = E_0 \left(\hat{\underline{\mathbf{x}}} \cos[k_0 z - \omega_0 t + \phi_0] + \hat{\underline{\mathbf{y}}} \cos\left[k_0 z - \omega_0 t + \phi_0 \pm \frac{\pi}{2}\right] \right)$$

In this case, the sum of electric vectors follows a circular path in time and space, and thus the wave is called *circularly polarized.* The angle of the electric field as a function of $[z,t]$ may be evaluated:

$$\begin{aligned}
\theta &= \theta[z,t] = \tan^{-1}\left[\frac{E_0 \cos\left[k_0 z - \omega_0 t + \phi_0 \pm \frac{\pi}{2}\right]}{E_0 \cos[k_0 z - \omega_0 t + \phi_0]}\right] = \tan^{-1}\left[\frac{\cos\left[k_0 z - \omega_0 t + \phi_0 \pm \frac{\pi}{2}\right]}{\cos[k_0 z - \omega_0 t + \phi_0]}\right] \\
&= \tan^{-1}\left[\frac{\mp \sin[k_0 z - \omega_0 t + \phi_0]}{\cos[k_0 z - \omega_0 t + \phi_0]}\right] \\
&= \tan^{-1}[\mp \tan[k_0 z - \omega_0 t + \phi_0]] = \mp[k_0 z - \omega_0 t + \phi_0]
\end{aligned}$$

In words, the angle of the electric vector is a linear function of z and of t; the angle of the electric vector changes with time and space.

You may want to reexamine or redo that experiment. A similar configuration where the amplitudes in the x- and y-directions are not equal is called *elliptically polarized.*



Propagation of circularly polarized light: the phase of E_y is delayed (or advanced) by $\frac{\pi}{2}$ radians relative to E_x.

Circularly polarized light may be generated by delaying the phase of one component of plane-polarized light by $\pi/2$ radians, or 1/4 of a period. The device for introducing the phase delay is called a *quarter-wave plate.* We can also construct a *half-wave plate* such that the output wave is:

$$\begin{aligned}
\underline{\mathbf{E}}[z,t] &= E_0 \left(\hat{\underline{\mathbf{x}}} \cos[k_0 z - \omega_0 t + \phi_0] + \hat{\underline{\mathbf{y}}} \cos[k_0 z - \omega_0 t + \phi_0 \pm \pi] \right) \\
&= E_0 \left(\hat{\underline{\mathbf{x}}} \cos[k_0 z - \omega_0 t + \phi_0] + \hat{\underline{\mathbf{y}}} (-\cos[k_0 z - \omega_0 t + \phi_0]) \right) \\
&= E_0 \left(\hat{\underline{\mathbf{x}}} - \hat{\underline{\mathbf{y}}} \right) \cos[k_0 z - \omega_0 t + \phi_0]
\end{aligned}$$

$$\theta = \theta[z,t] = \tan^{-1}\left[\frac{-E_0 \cos[k_0 z - \omega_0 t + \phi_0]}{E_0 \cos[k_0 z - \omega_0 t + \phi_0]}\right] = \tan^{-1}[-1] = -\frac{\pi}{4}$$

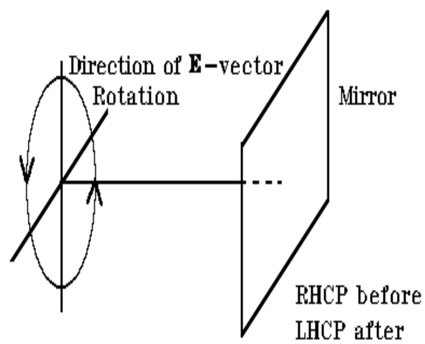
1.1.3 NOMENCLATURE FOR CIRCULAR POLARIZATION

Like linearly polarized light, circularly polarized light has two orthogonal states, *i.e.,* clockwise and counterclockwise rotation of the $\underline{\mathbf{E}}$-vector. These are termed *right-handed* (RHCP) and *left-handed* (LHCP). There are two conventions for the nomenclature:

1. Angular Momentum Convention (my preference): Point the thumb of the $\left\{ \begin{array}{c} \text{right} \\ \text{left} \end{array} \right\}$ hand in the direction of propagation. If the fingers point in the direction of rotation of the $\underline{\mathbf{E}}$-vector, then the light is $\left\{ \begin{array}{c} \text{RHCP} \\ \text{LHCP} \end{array} \right\}$.

2. Optics (also called the "screwy") Convention: The path traveled by the **E**-vector of RHCP light is the same path described by a right-hand screw. Of course, the natural laws defined by Murphy ensure that the two conventions are opposite: RHCP light by the angular momentum convention is LHCP by the screw convention.

The conservation of angular momentup ensures that the "handedness" of circularly or elliptically polarized light changes upon reflection, i.e., if the incident wave is RHCP, then the reflected wave is LHCP.



The "handedness" of circularly or elliptically polarized light changes upon reflection due to conservation of angular momentum.

1.2 Equipment:

1. Set of polarizers, including linear and circular polarizers, quarter- and half-wave plates
2. He:Ne laser
3. Fiber-optic light source
4. Radiometer (light meter), to measure the light transmitted by the polarizers;
5. Optical rail + carriers to hold polarizers, etc.
6. One rotatable stage to hold polarizer at measurable azimuth angle measured from vertical
7. Saran Wrap$^{\text{TM}}$ or Handi-Wrap$^{\text{TM}}$: stretchy wrap used for sandwiches

1.3 Procedure:

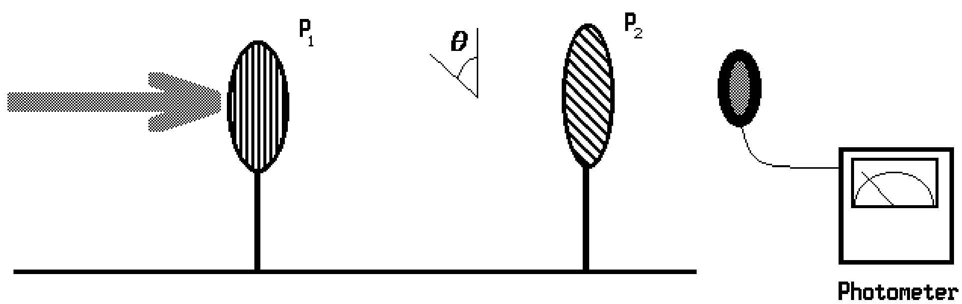
This lab consists of several sections:

1. an investigation of plane-polarized light, including a measurement of the intensity of light after passing through two polarizers oriented at a relative angle of θ
2. an investigation of Malus' law for plane-polarized light.
3. a mechanism for generating plane-polarized light by reflection,
4. an investigation of circularly polarized light, and
5. a demonstration of polarization created by scattering of the electric field by air molecules.

Your lab kit includes linear and circular polarizers, and quarter-wave and half-wave plates.

1. Plane-Polarized Light: the common mechanism for generating polarized light from unpolarized light is a "filter" that removes any light with electric vectors oriented perpendicular to the desired direction. This is the device used in common polarized sunglasses.
   - (a) Orient two polarizers in orthogonal directions and look at the transmitted light.
   - (b) Add a third polarizer AFTER the first two so that it is oriented at an angle of approximately $\pi/4$ radians (45°) and note the result.
   - (c) Add a third polarizer BETWEEN the first two so that it is oriented at an angle of approximately $\pi/4$ radians and note the result.
2. Malus' Law

   This experiment uses laser sources, and a few words of warning are necessary: **NEVER LOOK DIRECTLY AT A LASER SOURCE THE INTENSITY IN THE BEAM IS VERY CONCENTRATED AND CAN DAMAGE YOUR RETINA**



   - (a) Mount a screen behind polarizer P_1 and test the radiation from the laser to see if it is plane polarized and, if so, at what angle.
   - (b) Measure the "baseline" intensity of the source using the CCD camera (with lens). You may have to attenuate the light source with a piece of paper and/or stop down the aperture of the lens.
   - (c) Insert one polarizer in front of the detector and measure the intensity relative to the original unfiltered light. How much light does one polarizer let through? Next, increase the intensity of the light until you nearly get a saturated image on the CCD.
   - (d) Add a second polarizer to the path and orient it first to maximize and then to minimize the intensity of the transmitted light. Measure both intensities.

(e) Next we will try to confirm Malus' Law. Two polarizers must again be used: one to create linearly polarized light, and one to "test" for the polarization. The second polarizer often is called an analyzer, and if possible, use one of the rotatable polarizers on the optical mounts. Set up the polarizers and adjust the light intensity so that when they are aligned, the image on the camera is again nearly saturated.

(f) Measure the transmitted light for different relative angles (say, every 10° or so) by rotating the second polarizer. "Grab" an image of the source through the two polarizers and determine the average pixel value for your light source at each angle. Plot the data graphically as a function of angle and compare to Malus' law:

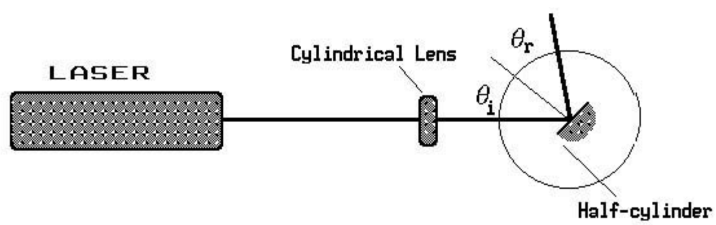
3. Polarization by reflection:

(a) Look at the reflection of light from a glossy surface (such as reflections of the ceiling lights from a waxed floor or the smooth top of a desk) through the polarizer. Rotate the polarizer to find the direction of greatest transmission. This indicates the direction of linear polarization that is transmitted by the polarizer. If possible, document the reflection at different angles by a photograph with a digital camera through the polarizer.

(b) Use a setup shown so that the laser beam reflects from a glass prism; the prism just serves as a piece of glass with a flat surface and not for dispersion or internal reflection. Place one of the linear polarizers between the laser and the first mirror. Focus your attention on the intensity of the beam reflected from the front surface of the prism – you probably will need to darken the room.

(c) Rotate the prism on the paper sheet with angular markings to vary the incident angle θ_i. At each θ_i, rotate the polarizer to minimize the transmitted intensity. At a particular θ_i, the intensity of the transmitted beam should be essentially zero. This angle θ_i of minimum reflection is Brewster's angle, where the electric-field vector parallel to the plane of incidence is not reflected. Measure Brewster's angle at least 4 times and average to get your final result.

(d) Remove the polarizer from the incident beam and use it to examine the state of polarization of the reflected light.



4. Circular Polarization: as stated above, circularly polarized light is generated by delaying (or advancing) the phase of one component of linearly polarized light by 90° with a quarter-wave plate. A circular polarizer can be constructed from a sandwich of a linear polarizer and a quarter-wave plate. In this way, the circular polarizer is sensitive to the direction of propagation of the light.

(a) Construct a circular polarizer with the components available and test it by laying it on a shiny surface (a coin works well). Shine a light source on the arrangement from above, and test the behavior as you rotate the linear polarizer. Make sure you can see a variation in intensity of the light reflected by the coin. You should be able to see one clear minimum in intensity between 0 and 90°.

(b) Orient your CCD camera so you can grab images of the coin at different orientation angles. Measure the intensity at several (∼7) different angles from 0 to 90 degrees, including one near the minimum intensity.

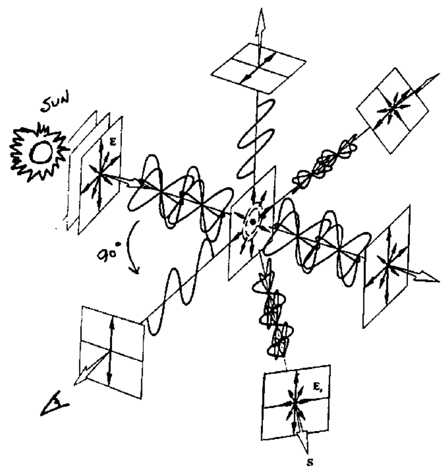
5. Make a quarter-wave plate from several (6 or 7) layers of sandwich wrap by taping the wrapping to a piece of cardboard with a hole cut in it. This is convenient because it can be "tuned" to specific colors of light by adding or subtracting layers (more layers for longer λ).. Test to see if this actually acts as a quarter-wave plate by using it to make a circular polarizer and testing the reflection from a shiny coin.

6. Polarization by Scattering (if outside sky is clear and blue) (Yeah, right, isn't this in Rochester?)

   (a) Examine scattered light from the blue sky for linear polarization. Look at several angles measured relative to the sun.

   (b) Determine the direction where the light is most completely polarized. This knowledge is useful to determine the direction of polarization of any linear polarizer.

   (c) Test skylight for circular polarization.



Polarization of sunlight due to scattering by molecules in the atmosphere.

1.4 Analysis:

In your writeups, be sure to include the following items.

1. Plot the expected curve for Malus' Law together with your experimental data.

2. State your final result for Brewster's angle with $\frac{\sqrt{N}}{\sigma}$ uncertainty.

3. Graph your results for the brightness of the coin as a function of the orientation angle of the linear polarizer. Explain why the minimum occurs where it does.

1.5 Questions:

1. Consider why sunglasses used while driving are usually made with polarized lenses. Determine the direction of polarization of the filters in sunglasses. The procedure to determine the direction of polarization of light reflected from a glossy surface or by scattering from molecules are helpful.

2. Explain the action of the circular polarizer on reflection.

3. Considered the circular polarizer you constructed. If the angle of the plane (linear) polarizer is not correct, what will be the character of the emerging light?