1051-455-20073, Physical Optics

1 LAB #1 – 1-D and 2-D HARMONIC OSCILLATIONS

1.1 Rationale:

This laboratory (really a virtual lab based on computer software) introduces the concepts of harmonic oscillations and the effects that result when multiple harmonic oscillations are superposed (added) in one and two dimensions. The 1-D case illustrates concepts that are relevant to temporal and spatial coherence, diffraction, and interference of electromagnetic radiation, though these more advanced applications will not be considered until later in the course. The 2-D case is a generalization of the description of oscillations using complex notation; $z \equiv a + ib$, where $[a, b]$ are real numbers and $i \equiv \sqrt{-1}$.

A "harmonic" oscillation is composed of a single sinusoidal frequency. Projections of 2-D harmonic oscillatory motion onto any radial line through the origin in the 2-D plane yields examples of 1-D harmonic oscillatory motion which all have the same frequency and amplitude, and with the initial phase determined by the particular axis chosen.

1.2 Preparation:

1. Write the general equation for a simple harmonic oscillation in trigonometric form (*i.e.,* as a function of the trigonometric functions sine, cosine, *etc.*).

2. Draw a diagram of the motion of a harmonic oscillator as a function of time, including a graph of the output as a function of time. Designate on the drawings the amplitude, period, angular frequency, phase angle, and initial phase, and specify the units for each.

1.3 Equipment:

1. Personal Computer running Windows

2. Java Program "*Signals.JAR*" (installed on laboratory computers) (the Java virtual machine must be installed, which is available from http://www.java.com/en/download.index.jsp)

3. "*Audacity.EXE*" (available free for IBM-compatible PCs, Mac OS X, and Linux at http://audacity.sourceforge.net/download/)

1.4 Procedure:

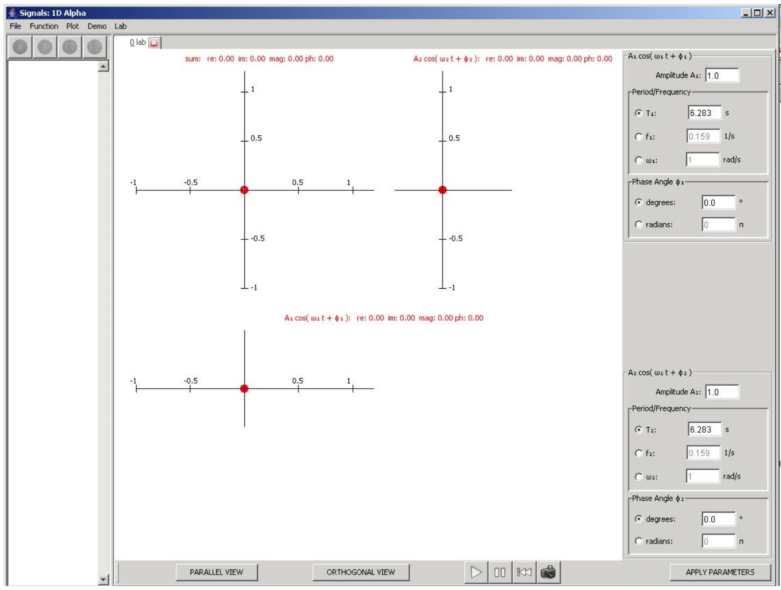
The lab is based on a program written in Java by Juliet Bernstein, an undergraduate student in the Chester F. Carlson Center for Imaging Science. It allows sinusoidal functions to be added and displayed dynamically. The graphics screens may be grabbed as ".png" or ".jpg" files and pasted into application programs (such as WORD^{TM} or POWERPOINT^{TM}) for lab reports.

1.4.1 Summation of 1-D oscillations

The summation of same-frequency and different-frequency oscillations will be investigated. To run the program:

1. Start the program by double-clicking on the desktop icon, which should be located in a desktop folder named "455 Optics Labs."

2. Click on the "lab" tab and select "Physical Optics Lab 1;" you will see this screen:



Opening page of Lab in "Signals.jar"

3. Select the parameters for the two functions: the amplitude A_n, the (approximate) period T_n in seconds (or the temporal frequency ν_n in Hz or the angular temporal frequency ω in $\frac{\text{radians}}{\text{sec}}$), and the initial phase ϕ_n (in degrees or in units of π radians). Note that the values may be negative. **Whenever you change a parameter, be SURE to hit the "APPLY PARAMETERS" button in the bottom right.**

$$A_1 \cos[\omega_1 t + \phi_1] = A_1 \cos[2\pi\nu_1 t + \phi_1] = A_1 \cos\left[2\pi \frac{t}{T_1} + \phi_1\right]$$

$$A_2 \cos[\omega_2 t + \phi_2] = A_2 \cos[2\pi\nu_2 t + \phi_2] = A_2 \cos\left[2\pi \frac{t}{T_2} + \phi_2\right]$$

4. Select the option for "parallel view where the oscillations are added in the same direction.

5. Set the temporal period of the two oscillations to be $T_1 = T_2 = 1\,\text{s}$ and the amplitudes to be identically $A_1 = A_2 = 1$ and the initial phase $\phi_1 = \phi_2 = 0$. Watch the show and use the stopwatch to measure the actual period. The "Pause" button ()stops the motion, which may be restarted by pushing "Play" ().

   (a) Repeat for $T_1 = T_2 = 2\,\text{s}$

   (b) Repeat for $T_1 = T_2 = 3\,\text{s}$

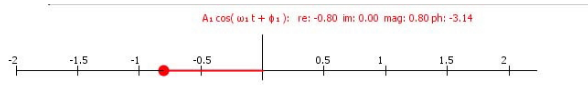
   (c) Repeat for $T_1 = T_2 = 4\,\text{s}$

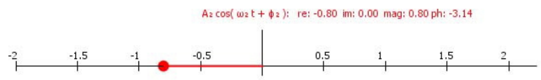
   (d) Repeat for $T_1 = T_2 = 5\,\text{s}$

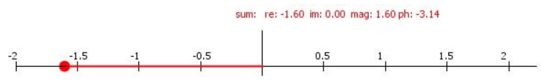
   (e) Repeat for $T_1 = T_2 = 10\,\text{s}$

   (f) Plot the relationship between the selected and measured temporal periods. Perform a *linear regression* to fit a line to the data; this may be done in a spreadsheet program like Excel. Record the *correlation coefficient* of the linear regression. Plot the regression line along with the data.

The graphics screen obtained after summing two oscillations is shown below: the input oscillations are shown in the first two rows and the superposition in the third row.





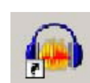


Observations:

1. Add two waves in parallel with the same angular frequency $\omega\ \left[\frac{\text{radians}}{\text{sec}}\right]$, different amplitudes A_1 and A_2 (both $A_n < 1$) and different initial phases ϕ_1 and ϕ_2. Measure the amplitude A and angular frequency ω of the resultant and compare to the relationship derived in class.

2. Repeat #2 for several different pairs of amplitudes and initial phases. Use to confirm the relationship for the sum of two same-frequency sinusoids with different amplitudes and phases.

3. Add two waves with the same amplitude $A_1 = A_2$ and similar (but not identical) angular frequencies ω_1 and ω_2. The resultant amplitude oscillates as a function of time in a complicated motion that actually is the sum of a rapidly varying term and a slowly varying term. Measure both temporal periods and compute the frequencies. This illustrates the important and common phenomenon of "BEATS", which appear in may areas of science.

1.4.2 Summation of 1-D oscillations using AUDACITY.EXE 

This is a simple demonstration of the summation of two waves to produce "beats." Open the program by double-clicking on the icon in the folder labeled "455 Optics Labs" or by using the Start button.

1. From the "Project" menu, pull down the option "new Audio Track." You can delete the track by clicking on the "X" in the upper-left corner. Note that you must **STOP** playing a track by pressing the stop button () before you can do any analysis (pressing the **PAUSE** button  is not sufficient)

2. From the "Generate" menu, pull down the option "Tone." The default is 440 Hz (the musical note "A" – the standard note for tuning instruments). Select an amplitude of 0.5, to prevent saturation when a second waveform of that amplitude is added. Play the tone by clicking on the green triangle just below the "Generate" menu.

3. From the "Project" menu, open a second audio track."

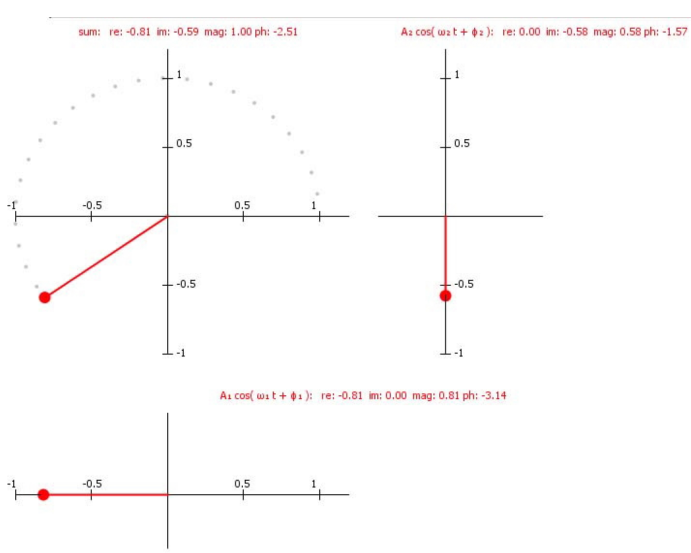
4. "From the "Generate" menu, load a sine wave with a frequency of 441 Hz.

5. Click on the "Play" button to play both tones simultaneously. What do you hear? Use a stopwatch to measure the period of the beat.
6. Examine the two individual waves by clicking on the magnifier to expand the scale.
7. Add the two waves:
   (a) "Select all" in the "Edit" menu
   (b) Select "Quick Mix" in the "Project" menu. This creates a single track that is the sum of both tracks
   (c) Expand the scale using the "Magnify" tool to examine the waveform.
8. Select a section of the sum waveform (say, about 5 s long) and choose "Plot Spectrum" from the "View" menu; this evaluates the fast Fourier transform of the selection and displays the spectrum, which is just the representation of the signal in terms of the amplitude A at each temporal frequency ν instead of in terms of the amplitude A at each time t. Since a true undistorted sinusoid $f[t] = A_0 \cos[2\pi\nu_0 t]$ is composed of only a single sinusoidal frequency ν_0, the ideal representation of its spectrum is a single "spike" with amplitude A_0 at the temporal frequency $\nu = \nu_0$.
9. Open a new track and generate a square wave of some base frequency. Listen to the tone and examine the spectrum. Describe the "quality" of the tone compared to the sinusoidal function.
10. Open a new track and generate "white noise;" listen to the signal and plot the spectrum. This should demonstrate why the noise is called "white."

1.4.3 Summation of Orthogonal 1-D oscillations using Signals

The summation of two orthogonal oscillations results in a *Lissajou* figure, which also is called an *Argand* diagram when imaginary numbers are plotted on the vertical axis. The two selected oscillations generate the resulting motion, but the system can considered in the other direction where the projections of the resulting motion along the x- and y-axis are the selected oscillations.

The graphics screen resulted from summation of two oscillations with the same angular frequency ω and amplitude A, but whose initial phases differed by $-\frac{\pi}{2}$ radians.



1. Orthogonally add pairs of oscillations to produce: linear vibration along the diagonal, circular motion in both the clockwise and counterclockwise directions, and elliptical motion where the major axis lies along one of the Cartesian axes (x or y), and an ellipse whose major axis is at another angle.
2. Change the periods to generate other kinds of figures; record by using the screen shot icon ().
3. If the two oscillations have the same frequency, but arbitrary amplitude and initial phase, what can you say about the result?

1.4.4 Summation of Arbitrary Orthogonal 1-D Functions using SIGNALS

1. First, replicate one of the cases already considered in the orthogonal summation by clicking on the "FUNCTION" menu and selecting "CREATE NEW FUNCTION" and "REAL and IMAGINARY PARTS" (you can get there via "CTRL-F")
   (a) Under "CREATE REAL PART," select "MORE" and a COSINE wave with a long period (say 128 samples in an array of size $N = 256$)
   (b) Under "CREATE IMAGINARY PART," select "MORE" and a COSINE wave with the same period and initial phase $\phi = -90°$
2. Display the function in the PLOT Menu as CUSTOM plots of the real+imaginary parts, magnitude+unwrapped phase, and as ARGAND diagram.
3. Reenter the arrays but change the initial phase of the imaginary part to a cosine wave with $\phi_0 = +90°$. What is the difference in the displayed Argand diagram?
   (a) We speak of the rate of change of the phase angle of the display as the frequency of the complex function. In this example, the rate of change of phase is constant, so the frequency is fixed.
4. Reenter the arrays but change the initial phase of the COSINE in the imaginary part to $\phi_0 = +45°$. What is the difference in the displayed Argand diagram?
5. Enter and view the new functions as real-imaginary parts, as magnitude/phase and as the ARGAND diagram.
   (a) REAL Part: COSINE with period = 64, amplitude = 1, center pixel = 0, initial phase = 0: IMAGINARY Part: COSINE with period = 64, amplitude = $\frac{1}{2}$, center pixel = 0, and initial phase = $-90° = -\frac{\pi}{2}$ radians.
   (b) REAL Part: COSINE with period = 64, amplitude = 1, center pixel = 0, initial phase = 0: IMAGINARY Part: COSINE with period = 48, amplitude = 1, center pixel = 0, and initial phase = $-90° = -\frac{\pi}{2}$ radians.
   (c) REAL Part: COSINE with period = 64, amplitude = 1, center pixel = 0, initial phase = 0: IMAGINARY Part: COSINE with period = 64, amplitude = 1, center pixel = 0, and initial phase = $-45° = -\frac{\pi}{4}$ radians.
   (d) REAL Part: COSINE with period = 64, amplitude = 1, center pixel = 0, initial phase = 0, IMAGINARY Part: COSINE with period = 64, amplitude = 1, center pixel = 0, and initial phase = $-135° = -\frac{3}{4}\pi$ radians.
   (e) REAL Part: CHIRP function with period = 64, amplitude = 1, center pixel = 0, and initial phase = 0, IMAGINARY Part: CHIRP with period = 64, amplitude = 1, center pixel = 0, and initial phase = $-90° = -\frac{\pi}{2}$ radians. What is different about the "frequency" of this function compared to that where the REAL and IMAGINARY parts are sinusoidal functions?

6. Repeat the procedure for a few other functions (your choice). For example, enter sinusoids of equal amplitudes and a phase difference of $\pm 90°$, and modulate the function by multiplying the entire function by a decaying function such as a negative exponential. View the Argand diagram (option "N" in the "PLOT" menu).

1.4.5 Nonlinear Operations on Sinusoidal Waves

In all of the steps thus far, the output has been the a single wave or the summation of two or more sinusoidal waves. In this section, we briefly consider the result of a specific "nonlinear" operation on a wave.

1. Enter a "bipolar" sinuoidal function into one of the arrays ("bipolar" means that the function has positive and negative amplitudes), e.g., $f[n] = \cos\left[2\pi \frac{x}{X_0}\right]$; this function oscillates between amplitudes of ± 1 and its average amplitude is 0. This function may be written in the complex form:

$$\begin{aligned} f[x] &= \frac{1}{2}\exp\left[i\cdot\left(2\pi\frac{x}{X_0}\right)\right] + \frac{1}{2}\exp\left[-i\cdot\left(2\pi\frac{x}{X_0}\right)\right] \\ &= \frac{1}{2}\exp\left[i\cdot\left(2\pi\left(+\xi_0\right)x\right)\right] + \frac{1}{2}\exp\left[i\cdot\left(2\pi\left(-\xi_0\right)x\right)\right] \end{aligned}$$

where $\xi_0 \equiv (X_0)^{-1}$ and $i \equiv \sqrt{-1}$. We say that this sinusoid is the sum of two complex sinusoids: one with spatial frequency $+\xi_0$ and one with frequency $-\xi_0$.

2. Now plot the square of $f[n]$. You can do this by duplicating the array (in the operations menu obtained via CTRL-O) and then multiplying them together (again in the operations menu).

$$(f[x])^2 = (\cos[2\pi\xi_0 x])^2$$

but we can rewrite this using the well-known identity:

$$\begin{aligned} (\cos[\theta])^2 &= \frac{1}{2}(1+\cos[2\theta]) \\ &\Longrightarrow (\cos[2\pi\xi_0 x])^2 = \frac{1}{2}(1+\cos[2\cdot 2\pi\xi_0 x]) \\ (\cos[2\pi\xi_0 x])^2 &= \frac{1}{2} + \frac{1}{2}\cos[2\pi(2\cdot\xi_0)x] \\ &= \frac{1}{2} + \frac{1}{2}\left(\frac{1}{2}\exp\left[i\cdot\left(2\pi\left(+2\xi_0\right)x\right)\right] + \frac{1}{2}\exp\left[i\cdot\left(2\pi\left(-2\xi_0\right)x\right)\right]\right) \\ &= \frac{1}{2} + \frac{1}{4}\exp\left[i\cdot\left(2\pi\left(+2\xi_0\right)x\right)\right] + \frac{1}{4}\exp\left[i\cdot\left(2\pi\left(-2\xi_0\right)x\right)\right] \end{aligned}$$

This is composed of a cosine with infinite period and amplitude $\frac{1}{2}$ and two complex-valued sinusoids that oscillate with frequencies $\pm 2\cdot\xi_0$ with amplitude $\frac{1}{4}$. In words, the nonlinear operation has "created" some different spatial frequencies. This did not happen if signals were just added together.

1.5 Questions:

1. Consider two travelling waves:

$$\begin{aligned} y_1(t) &= A_1\cos\left[\frac{2\pi x}{\lambda_1} - 2\pi\nu_1 t\right] \\ y_2(t) &= A_1\sin\left[\frac{2\pi x}{\lambda_1} + 2\pi\nu_1 t + \frac{\pi}{3}\right] \end{aligned}$$

Find an expression for the sum of the two waves and sketch it as a function of x in the interval $-3\lambda_1 \leq x \leq 3\lambda_1$ for times $t = 0, \frac{1}{2\nu_1}, \frac{1}{\nu_1}$, and $\frac{5}{\nu_1}$. Label the zero crossings, *i.e.,* the coordinates x where the wave has zero amplitude.

2. Repeat question 1 for the two waves:

$$\begin{aligned} y_3(t) &= A_1 \cos\left[\frac{2\pi x}{\lambda_1} - 2\pi\nu_1 t\right] \\ y_4(t) &= A_1 \cos\left[\frac{2\pi x}{\lambda_1} + 2\pi\nu_1 t\right] \end{aligned}$$

   and for the waves

$$\begin{aligned} y_5(t) &= A_1 \cos\left[\frac{2\pi x}{\lambda_1} - 2\pi\nu_1 t\right] \\ y_6(t) &= \frac{A_1}{2} \cos\left[\frac{2\pi x}{\lambda_1} + 2\pi\nu_1 t\right] \end{aligned}$$

3. Explain the difference in tone "quality" in Audacity between the sine and square waves.