SIMG-455-20073 Solution Set #4

1. Find the phase difference between light that is reflected normally (angle of incidence $\theta_0 = 0$) from the inner and from the outer surfaces of a film of magnesium fluoride (MgF, $n = 1.38$) deposited on the surface of a lens ($n = 1.5$) if the film thickness is $t = 100\,\text{nm}$ for violet light with $\lambda_0 = 400\,\text{nm}$ and for red light with $\lambda_0 = 700\,\text{nm}$; assume that the indices are identical for both wavelengths.

We need to determine the phase difference of light with $\lambda_0 = 400\,\text{nm}$ *and for red light with* $\lambda_0 = 700\,\text{nm}$ *after it reflects from the front and back surfaces of a thin film of MgF with* $n = 1.38$*. The phase difference includes differences from the phase shifts on reflection and from the optical path length difference. The reflections of the incoming light at both the front and back surfaces of the thin films are "rare-to-dense" and thus have no intrinsic phase change. The light that reflects from the second surface travels twice the thickness of the layer, so we need to find the numbers of wavelengths in twice the thickness of the film at the two wavelengths. For wavelength* λ_0 *in vacuum, the wavelength in a material with refractive index* $n = 1.38$ *is:*

$$\lambda' = \frac{\lambda_0}{n} = \frac{\lambda_0}{1.38}$$

$$\lambda_0 = 400\,\text{nm} \Longrightarrow \lambda' = \frac{400\,\text{nm}}{1.38} \cong 289.86\,\text{nm}$$

$$\lambda_0 = 700\,\text{nm} \Longrightarrow \lambda' = \frac{700\,\text{nm}}{1.38} \cong 507.25\,\text{nm}$$

The numbers of waves in thickness $2 \times t = 200\,\text{nm}$ *are:*

$$n_{400} = \frac{200\,\text{nm}}{289.86\,\text{nm}} \cong 0.690$$

$$n_{700} = \frac{200\,\text{nm}}{507.25\,\text{nm}} \cong 0.394$$

The optical phase differences between the light reflected from the first and second surfaces for the two wavelengths are:

$$\Delta\phi_{400} = 2\pi \bullet n_{400} = 2\pi \bullet 0.690 \cong 1.38\pi \text{ radians} \cong \boxed{248.4^\circ \cong \Delta\phi_{400}}$$

$$\Delta\phi_{700} = 2\pi \bullet n_{700} = 2\pi \bullet 0.394 \cong 0.78\pi \text{ radians} \cong \boxed{140.4^\circ \cong \Delta\phi_{700}}$$

2. Consider a two-aperture experiment with $d = 0.1\,\text{mm}$ and the distance to the observation screen $L = 500\,\text{mm}$; compute the distance between adjacent maxima on the screen for $\lambda_0 = 400\,\text{nm}$ and $\lambda_0 = 700\,\text{nm}$.

$$D \cong \frac{L\lambda_0}{d} = \frac{500\,\text{mm}}{0.1\,\text{mm}}\lambda_0 = 5000\lambda_0$$

$$D_{400} \cong 5000 \cdot 400\,\text{nm} = 2 \cdot 10^6\,\text{nm} = \boxed{2\,\text{mm} \cong D_{400}}$$

$$D_{700} \cong 5000 \cdot 700\,\text{nm} = 3.5 \cdot 10^6\,\text{nm} = \boxed{3.5\,\text{mm} \cong D_{700}}$$

3. Determine the number of fringe cycles per millimeter if the angle between two plane waves of light is 5° and $\lambda_0 = 632.8\,\text{nm}$.

$$\sin\left[\frac{\theta}{2}\right] = \frac{\left(\frac{d}{2}\right)}{L} \Rightarrow \frac{d}{L} \cong 2 \cdot \sin\left[\frac{\theta}{2}\right]$$

$$D \cong \frac{L\lambda_0}{d} = \frac{\lambda_0}{2 \cdot \sin[\frac{\theta}{2}]} = \frac{632.8\,\text{nm}}{2 \cdot \sin[2.5°]} \cong 7.25\,\mu\text{m}$$

The number of fringe cycles per millimeter is the spatial frequency; the reciprocal of the period:

$$\frac{1}{D} = \frac{1}{7.25\,\mu\text{m}} \cong \boxed{0.138\,\frac{\textit{cycles}}{\mu\text{m}} = 138\,\frac{\textit{cycles}}{\text{mm}}}$$

4. Consider a two-aperture experiment with the following characteristics: $\lambda_0 = 550\,\text{nm}$, $d = 3.3\,\text{mm}$, $L = 3\,\text{m}$:

a. Calculate the fringe spacing;

$$D \cong \frac{L\lambda_0}{d} = \frac{3\,\text{m} \cdot 550\,\text{nm}}{3.3\,\text{mm}} = \boxed{0.5\,\text{mm} \cong D}$$

b. Place a sheet of glass with plane-parallel faces and thickness $\tau = 10\,\mu\text{m}$ in front of one of the apertures; determine the direction of displacement of the fringes and derive the formula giving the relationship for their displacment

The light through the aperture covered by the glass travels more slowly through the glass and thus travels a longer optical path than the light through the uncovered aperture. Consider the fringe at the center if both apertures are uncovered, which means that the optical path difference at this fringe is zero; if one aperture is then covered, the fringe where the path length difference is zero must move in the direction of the aperture with the glass (i.e., if the upper aperture is covered by glass, the zero-path-difference fringe moves "up" so that the physical path through the uncovered aperture is longer than the path through the covered aperture, while the optical path lengths match. If the optical path from the aperture to the location x on the observation screen is

$$\ell = \frac{x}{D} \cdot \lambda_0$$

and the optical path through the glass is:

$$OPL_{glass} = n \cdot \tau$$

then the total optical path length from the aperture through the glass to the observation screen is:

$$OPL = \ell - t + n \cdot t = \ell + (n-1) \cdot \tau$$
$$= \frac{x}{D} \cdot \lambda_0 + (n-1) \cdot \tau$$

The optical phase difference is the number of wavelengths multiplied by 2π radians per cycle:

$$O\Phi D = \frac{2\pi}{\lambda_0} \cdot \left(\frac{x}{D} \cdot \lambda_0 + (n-1) \cdot \tau\right)$$
$$= 2\pi \cdot \frac{x}{D} + \frac{2\pi}{\lambda_0}(n-1) \cdot \tau$$

If the phase difference is zero, the new coordinate x is:

$$2\pi \cdot \frac{\Delta x}{D} + \frac{2\pi}{\lambda_0}(n-1) \cdot \tau = 0$$
$$\frac{\Delta x}{D} = -\frac{1}{\lambda_0}(n-1) \cdot \tau$$
$$\Rightarrow \boxed{\Delta x = -\frac{D}{\lambda_0}(n-1) \cdot \tau}$$

The negative sign indicates that the fringe moves towards the glass.

c. In part (b), if the fringes are displaced by $\Delta x = 4.73\,\text{mm}$, determine the refractive index of the glass:

To find the refractive index, solve for n:

$$n = 1 + \frac{\Delta x \cdot \lambda_0}{D \cdot \tau}$$

$$= 1 + \frac{4.73\,\text{mm} \cdot 550\,\text{nm}}{0.5\,\text{mm} \cdot 10\,\mu\text{m}} = \boxed{n = 1.52}$$

d. (optional, bonus) if the error in the measurement of the fringe displacement is $\pm 0.01\,\text{mm}$, determine the error in the measurement of *n*.

$$n = 1 + \frac{\Delta x \cdot \lambda_0}{D \cdot \tau} \Rightarrow dn = n = \frac{\lambda_0}{D \cdot \tau} \cdot d(\Delta x)$$

$$= \frac{(n-1)}{\Delta x} \cdot d(\Delta x)$$

$$= \frac{(1.52 - 1)}{4.73\,\text{mm}} \cdot (0.02\,\text{mm})$$

$$\cong 2 \times 10^{-3}$$

$$\Rightarrow n = 1.52 \pm 0.002$$

5. If one mirror of a Michelson interferometer is translated by some distance, 200 fringes are observed to pass a specific point in the field of view. If $\lambda_0 = 624\,\text{nm}$, determine the translation distance of the mirror.

Call the translation distance ΔL*, so the optical path difference is* $2 \cdot \Delta L$ *and the optical phase difference is:*

$$\Delta\Phi = \frac{2\pi}{\lambda_0} \cdot (2 \cdot \Delta L)$$

A phase difference of $\Delta\Phi = 2\pi \Rightarrow$ *one fringe, so that 200 fringes implies a phase change of* 400π*:*

$$400\pi = \frac{4\pi}{\lambda_0} \cdot \Delta L \Rightarrow \Delta L = 100\lambda_0 = 100 \cdot 624\,\text{nm} = \boxed{\Delta L = 62.4\,\mu\text{m} = 0.0624\,\text{mm}}$$

6. A mercury source is positioned behind a bandpass filter that removes the ultraviolet portion and transmits only green light with $\lambda = 546.1\,\text{nm}$, The light is allowed to pass through a narrow horizontal slit positioned $1\,\text{mm}$ above a flat mirror surface. Describe both qualitatively and quantitatively what appears on a screen located $1\,\text{m}$ away from the slit.

This is the so-called "Lloyd's mirror" experiment where the reflection of the light through the slit creates a virtual image of a second slit. Since the real slit is positioned at a distance $1\,\text{mm}$ *above the mirror, the apparent distance between the real and virtual slits is* $d = 2\,\text{mm}$. *Note that the reflected light exhibits a phase change of* π *radians, so that the optical phase difference of the light is:*

$$\Delta\Phi = \frac{2\pi}{\lambda_0} \cdot \frac{Dd}{L} + \pi$$

The period of the fringe pattern is:

$$D \cong \frac{L\lambda_0}{d} = \frac{1\,\text{m} \cdot 546.1\,\text{nm}}{2\,\text{mm}} = \boxed{D = 0.273\,\text{mm}}$$

and there is a fringe minimum at the mirror, so that the fringe maxima are located where $\Delta\Phi = 2\pi m$*, where m is an integer:*

$$2\pi m = \frac{2\pi}{\lambda_0} \cdot \frac{Dd}{L} + \pi \Rightarrow m = \frac{Dd}{\lambda_0 L} + \frac{1}{2}$$

The irradiance pattern is:

$$I[y] = 4I_0 \cos^2\left[2\pi \frac{y}{D} + \pi\right] = 4I_0 \sin^2\left[2\pi \frac{y}{0.273\,\text{mm}}\right]$$