# 1051-455-20073 Homework #6

Read $P^3$ §9 *Coherence* and $P^3$ §11 *Fraunhofer Diffraction*

**Due 16 May 2008 (Th) –:** Write up ONE lab of your choice in the "long format" (including abstract, data, analysis, etc., as listed in the handout)

**Due 8 May 2008 (Th)** – Do the following problems; SHOW YOUR WORK

1. "White" light includes equal "amounts" of each wavelength in the interval $400\,\text{nm} \leq \lambda \leq 700\,\text{nm}$.
   (a) Determine the frequency bandwidth for this wavelength range
   (b) Compute the associated *coherence time* and *coherence length* of white light.

2. The range of angular temporal frequencies by a light source is $\Delta\omega$:
   (a) Find the expression for $\Delta\nu$
   (b) Derive the expression for the corresponding *linewidth* $\Delta\lambda$
   (c) Derive the expression for the *coherence length* $\Delta\ell$ of the source.
   (d) Find the coherence length of a sodium arc that emits two narrow spectral lines:

$$\omega_1 = 3.195 \times 10^{15} \frac{\text{radians}}{\text{sec}}$$
$$\omega_2 = 3.198 \times 10^{15} \frac{\text{radians}}{\text{sec}}$$

   (e) Find the coherence length of a He:Ne "greenie" laser with

$$\omega_1 = 3.171 \times 10^{15} \frac{\text{radians}}{\text{sec}}$$
$$\omega_2 = 3.469 \times 10^{15} \frac{\text{radians}}{\text{sec}}$$

3. Determine the linewidth in nanometers and in Hertz for laser light whose coherence length is 10 km if the mean wavelength is 632.8 nm (He:Ne)

4. Michelson found that the cadmium red line ($\lambda_0 = 643.8\,\text{nm}$) was the best available light source for his interference experiment. With it, he could see fringes for optical path differences up to 300 mm. Estimate the linewidth $\Delta\lambda$ and coherence time $\Delta t$ of this light source.

5. A light source emits two wavelengths $\lambda_1$ and $\lambda_2$. The light is incident upon a binary (composed of regions that are perfectly transparent or perfectly opaque) $f[x,y]$. The light then propagates to an observation screen located at a very large distance $L$ from the object. Describe and give reasons for the *qualitative* appearanceof the observed patterns for the following objects; you may also describe the patterns quantitatively for extra credit.
   (a) $f_a[x,y]$ is a single very small transparent aperture ("hole")
   (b) $f_b[x,y]$ consists of two apertures that are very narrow along the $x-$axis and infinitely long along the $y-$axis and that are separated by $d$ units.
   (c) $f_c[x,y]$ consists of an infinite number of apertures from part $b$ that are uniformly spaced at increments of $d$ units.

MORE→→→

6. The light diffracted by an object of the form $f[x, y]$ and observed at a distance $z_1$ in the Fraunhofer diffraction region has the "shape" of the squared magnitude of the Fourier transform of the object after appropriate rescaling of the coordinates back to the space domain

$$g[x, y] \propto |F[\xi, \eta]|^2 \Big|_{\xi \to \frac{x}{\lambda_0 z_1}, \eta \to \frac{y}{\lambda_0 z_1}}$$

where the 2-D Fourier transform is defined:

$$F[\xi, \eta] \equiv \mathcal{F}_2 \{f[x, y]\} \equiv \iint_{-\infty}^{+\infty} f[x, y] \quad \exp[-2\pi i (\xi x + \eta y)] \; dx \; dy$$

The object $f[x, y]$ satisfies the following conditions:

$$\begin{array}{ccc} f[x, y] = 1 & \text{if} & |x| \leq 1 \text{ AND } |y| \leq 1 \\ f[x, y] = 0 & f[x, y] = 1 & \text{otherwise} \end{array}$$

(a) Sketch $f[x, y]$;

(b) Calculate the diffraction pattern in the Fraunhofer diffraction region if $f[x, y]$ is illuminated by light with wavelength $\lambda_0$;

(c) Sketch the x-axis profile of the diffraction pattern including labels of the values on both axes.