1051-455-20073 Homework #3 Due 7 April 2007 (M)

Read for meaning P^3 §14 *Matrix Treatment of Polarization and* P^3 §15 *Production of Polarized Light*
Do the following problems, **SHOW YOUR WORK**!

1. The variation of refractive index with wavelength for a transparent substance (such as glass) may be approximately represented by the empirical equation due to Cauchy:

$$n[\lambda] \cong A_0 + \frac{B_0}{\lambda_0^2}$$

where A_0 and B_0 are empirically determined constants and λ_0 is the wavelength of light in a vacuum. If $A_0 = 1.40$, $B_0 = 2.5 \cdot 10^4\,(\text{nm})^2$, determine the phase and group velocities at $\lambda_0 = 500\,\text{nm}$.

2. For the crown and flint glasses given in the notes with the following indices measured at two vacuum wavelengths:

Line	λ_0 [nm]	n for Crown	n for Flint
C	656.28	1.51418	1.69427
F	486.13	1.52225	1.71748

Approximate the empirical constants A_0 and B_0 in Cauchy's equation (given in #1) and use them to evaluate the refractive index at $\lambda_0 = 589.59\,\text{nm}$ (Fraunhofer's "D" line); compare the results to the actual values:

Line	λ_0 [nm]	n for Crown	n for Flint
D	589.59	1.51666	1.70100

3. The electric vector of a wave is given by the real expression:

$$\underline{\mathbf{E}}[z,t] = E_0\left[\hat{\underline{\mathbf{x}}}\cos[k_0 z - \omega_0 t] + \hat{\underline{\mathbf{y}}} \cdot b\cos[k_0 z - \omega_0 t + \phi_0]\right]$$

Show that this is equivalent to the complex-valued expression:

$$\underline{\mathbf{E}}[z,t] = E_0\left(\hat{\underline{\mathbf{x}}} + \hat{\underline{\mathbf{y}}} \cdot b\exp[+i\phi_0]\right)\exp[+i(k_0 z - \omega_0 t)]$$

4. Sketch diagrams to show the type of polarizations in #3 for the following cases:
   (a) $\phi_0 = 0, b = 1$
   (b) $\phi_0 = 0, b = 2$
   (c) $\phi_0 = +\frac{\pi}{4}, b = -1$

5. Write down the Jones vectors for the three cases in the previous problem.

6. For the following three Jones vectors:

$$\underline{\mathcal{E}}_1 = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}, \underline{\mathcal{E}}_2 = \begin{bmatrix} +i \\ -1 \end{bmatrix}, \underline{\mathcal{E}}_3 = \begin{bmatrix} 1-i \\ 1+i \end{bmatrix}$$

   (a) Determine the type of polarization of each wave;
   (b) Find Jones vectors that are orthogonal to each of the three cases and describe the state of polarization.

7. Initially unpolarized light passes in turn through three linear polarizers with transmission axes at 0°, 30°, and 60°, respectively, relative to the horizontal axis. What is the irradiance of the product light expressed as a percentage of the unpolarized light irradiance?

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8. Since a sheet of Polaroid is not an ideal polarizer, not all the energy of the $\underline{\mathbf{E}}$-vibrations parallel to the TA are transmitted, nor are all $\underline{\mathbf{E}}$-vibrations perpendicular to the transmission axis are absorbed. Suppose an energy fraction α us transmitted in the first case and a fraction β is transmitted in the second.

   (a) Extend Malus' law by calculating the irradiance transmitted by a pair of such polarizers with angle θ between their transmission axes. Assume initially unpolarized light of irradiance I_0. Show that Malus' law follows in the limit of the ideal case.

   (b) Let $\alpha = 0.95$ and $\beta = 0.05$ for a given sheet of Polaroid. Compare the transmitted irradiance with that of an ideal polarizer when initially unpolarized light is passed through two such sheets that have a relative angle between transmission axes of $0°$, $30°$, $45°$, and $90°$.