

Read (for meaning, i.e., don't "skim") P³ §5 *Superposition of Waves*

1. For the following two harmonic waves:

(a) Show on a phasor diagram:

$$\begin{aligned} f_1 [t] &= 2 \cos [\omega_0 t] \\ f_2 [t] &= 7 \cos \left[\omega_0 t - \frac{\pi}{4} \right] \end{aligned}$$

(b) Find the mathematical expression for the superposition $f_1 [t] + f_2 [t]$ in the form of a cosine.

2. Two plane waves of the same frequency that vibrate along the z -direction are:

$$\begin{aligned} f_1 [x, t] &= A_1 \cos \left[2\pi \left(\frac{x}{X_1} - \nu_1 t + \pi \right) \right]; A_1 = 40 \text{ mm}, X_1 = 30 \text{ mm}, \nu_1 = 20 \text{ Hz} \\ f_2 [y, t] &= A_2 \cos \left[2\pi \left(\frac{y}{Y_2} - \nu_2 t + \pi \right) \right]; A_2 = 20 \text{ mm}, Y_2 = 40 \text{ mm}, \nu_2 = 20 \text{ Hz} \end{aligned}$$

Evaluate the resultant waveform $f_1 [x, t] + f_2 [y, t]$ at $[x, y] = [50 \text{ mm}, 20 \text{ mm}]$

3. Consider the superposition of two sinusoidal traveling waves:

$$\begin{aligned} f_1 [z, t] &= A_1 \cos [k_1 z - \omega_1 t], \\ A_1 &= 101 \text{ mm}, \nu_1 = 100 \text{ Hz}, v_1 = 250 \frac{\text{m}}{\text{s}} \\ f_2 [z, t] &= A_2 \cos [k_2 z - \omega_2 t], \\ A_2 &= 99 \text{ mm}, \nu_2 = 150 \text{ Hz}, v_2 = 500 \frac{\text{m}}{\text{s}} \end{aligned}$$

(a) Find an expression for the resulting wave in terms of the average wave, the modulation wave, plus any remaining amplitude.

(b) Calculate the wavelengths of the average and modulation waves.

(c) Find the velocities of the average and modulation waves.

(d) Does this system exhibit normal or anomalous dispersion?

4. The phase velocity of waves in some medium is proportional to $\omega^{+\frac{1}{2}}$. Find an expression for the modulation velocity and determine whether the waves exhibit normal or anomalous dispersion.

5. Plot and write the equation of the superposition of the following harmonic waves:

$$\begin{aligned} E_1 &= \sin \left[\frac{\pi}{18} - \omega t \right] \\ E_2 &= 3 \cos \left[\frac{5\pi}{9} - \omega t \right] \\ E_3 &= 2 \sin \left[\frac{\pi}{6} - \omega t \right] \end{aligned}$$

where the period of each is 2 s.

6. A laser emits a monochromatic beam of wavelength λ_0 , which is reflected normally from a plane mirror that recedes from the light source at velocity v .

(a) Determine the beat frequency between the incident and reflected light.

(b) Determine the beat frequency between the incident and reflected light if the light is incident on the plane mirror at angle θ .