1051-455-20073 Homework #1 Due 3/20/2008 (Th)

WITH CORRECTION TO #6

Read P^3 §4 *Wave Equations* and skim §5 *Superposition of Waves*
Do the following problems: **SHOW YOUR WORK!**

1. Determine which of the following describe traveling waves. Where appropriate, draw the profile and find the speed and direction of motion.

   (a) $\psi[y,t] = \exp[[-(a^2y^2 + b^2t^2 - 2abty)]$

   (b) $\psi[z,t] = A\sin[az^2 - bt^2]$

   (c) $\psi[x,t] = A\sin\left[2\pi\left(\frac{x}{a}+\frac{t}{b}\right)^2\right]$

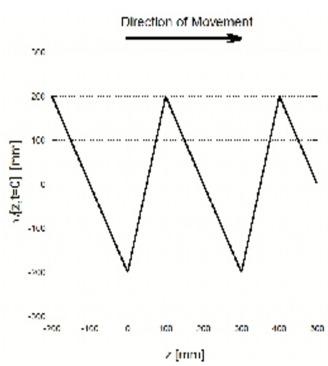
   (d) $\psi[x,t] = A\cos^2[2\pi(t-x)]$

2. The figure shows the profile of a transverse wave on a string traveling in the positive z-direction at a speed of $1\,\frac{\text{m}}{\text{s}}$.

   (a) Determine its wavelength.

   (b) Notice that as the wave passes any fixed point on the z-axis, the string at that location oscillates in time. Draw a graph of $\psi[t]$ showing how a point on the rope at $z=0$ oscillates.

   (c) What is the temporal frequency of the wave?



3. A particle executing simple harmonic motion given by

$$y[t] = 4\sin\left[2\pi\frac{t}{6}+\phi_0\right]$$

   is displaced by +1 unit when $t=0$. Find:

   (a) the phase angle $\Phi[t=0] \equiv \phi_0$

   (b) the difference in phase between any two positions of the particle separated in time by 2 sec;

   (c) the phase angle corresponding to a displacement of +2;

   (d) the time necessary to reach a displacement of +3 units from the initial position.

4. A wave vibrates according to the equation

$$y\left[z,t\right]=\frac{1}{2}\sin\left[\frac{\pi z}{3}\right]\cdot\cos\left[40\pi t\right]$$

where y and z are expressed in mm and t in sec.

(a) What are the amplitudes and the velocities of the component waves that give rise to this vibration?

(b) What is the distance between the nodes?

(c) What is the velocithy of a particle ath the position $z=1.5\,\mathrm{mm}$ when $t=\frac{9}{8}$ sec?

5. By finding appropriate relations for $\underline{\mathbf{k}}\bullet\underline{\mathbf{r}}$, write equations describing a sinusoidal plane wave in three directions in terms of wavelength and velocity for the three cases:

(a) propagation along the x-axis;

(b) propagation along the line $x=y$; $z=0$;

(c) propagation perpendicular to the planes $x+y+z=k$ where k is a constant.

6. Two waves of the same amplitude, speed, and frequency travel together in the same region of space. The resultant wave may be written as a sum of two individual waves: (**NOTE CORRECTION)**

$$\psi\left[z,t\right]=A_0\sin\left[k_0z+\omega_0t\right]+A_0\sin\left[k_0z-\omega_0t+\pi\right]$$

With the help of complex exponentials, show that:

$$\psi\left[z,t\right]=2A_0\cos\left[k_0z\right]\cdot\sin\left[\omega_0t\right]$$