

## Radiometry HW Problems<sup>1</sup>

**Problem 1.** Your night light has a radiant flux of 10 watts, what is the irradiance on your radiometry notes which fell 2 meters from the light when you fell asleep (assuming your notes were perpendicular to the night light)?

**Solution:**

We don't know the area of the notes but can at least propagate the flux out to the surface of a sphere of radius of 2 *m* (using point source approximations).

$$E_{sphere} \triangleq \frac{\partial \Phi_s}{\partial A} = \frac{10 \text{ W}}{4\pi r^2} = \frac{10 \text{ W}}{4\pi(2 \text{ m})^2} = \boxed{0.199 \text{ [W m}^{-2}\text{]}}$$

or

$$E = \frac{I_s}{r^2} = \frac{\partial \Phi_s / \partial \Omega}{r^2} = \frac{\partial \Phi_s}{\partial \Omega r^2} = \frac{10 \text{ W}}{4\pi(2 \text{ m})^2} = \boxed{0.199 \text{ [W m}^{-2}\text{]}}$$

**Problem 2.** Given the same 10 watt source above, what is the flux on your notes if you are using a standard sheet of paper measuring 8.5 *in* by 11 *in* and are 2 meters away (still perpendicular to the night light)? Additionally, what is the flux if the notes are tilted 25 degrees?

**Solution:**

Here, we subtend a section of the sphere with the area of the paper. At this point the irradiance onto the paper,  $E_{paper}$  is the same as the

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<sup>1</sup>Submit suggestions and typos to emmett@cis.rit.edu

irradiance on the surface of the sphere  $E_{sphere}$  of radius  $r$ .

$$\begin{aligned}
 E_{paper} &\triangleq \frac{\partial\Phi_{paper}}{\partial A_{paper}} \\
 \partial\Phi_{paper} &= E_{sphere} \partial A_{paper} = (0.199 \text{ W m}^{-2})(8.5 \text{ in})(11 \text{ in}) \\
 \partial\Phi_{paper} &= (0.199 \text{ W m}^{-2})(0.0603 \text{ m}^2) \\
 \partial\Phi_{paper} &= \boxed{12 \text{ mW}}
 \end{aligned}$$

If the notes are tilted 25 degrees, we have to account for the projected area. In many cases the angle is zero (*i.e.*, paper not tilted) and therefore  $\cos\theta = 1$ .

$$\partial\Phi_{tilt} = \partial\Phi_{paper} \cos\theta = (0.012 \text{ W}) \cos 25 = \boxed{10.9 \text{ mW}}$$

**Problem 3.** The camera you are using to record the CIS fall picnic has a detector with an area of  $2 \text{ cm}^2$  and a responsivity of  $2 \text{ VW}^{-1}$ . After all of the optics, the detector receives an irradiance of  $2 \text{ Wm}^{-2}$ . What is the output of the cell?

*Solution:*

$$\begin{aligned}
 \beta &\triangleq \frac{\partial S}{\partial\Phi} \\
 \partial S &= \beta \partial\Phi = \beta E \partial A \\
 S &= (2 \text{ VW}^{-1})(2 \text{ Wm}^{-2})(2 \text{ E}^{-4}\text{m}^2) \\
 S &= 8 \times 10^{-4} \text{ [V]} = \boxed{0.8 \text{ mV}}
 \end{aligned}$$

**Problem 4.** Your headlights have a radiant intensity of  $60 \text{ Wsr}^{-1}$ . Determine the irradiance on a sign 2 meters away.

*Solution:*

We can use the inverse square law as long as our distance from the source is at least ten times greater than the diameter of the source

(i.e.,  $\frac{dist}{dia} \gg 10$ ). For this problem, the diameter of the source can be no larger than

$$\begin{aligned} dia &<< \frac{dist}{10} \\ &<< \frac{2m}{10} = 20 \text{ cm} \end{aligned}$$

which is probably a little larger than a typical headlight, though pretty close. We certainly wouldn't want to use the inverse square law if we were less than 2 meters from the source. The solution is therefore,

$$E = \frac{I}{r^2} = \frac{60 \text{ Wsr}^{-1}}{(2 \text{ m})^2} = \boxed{15 \text{ [Wm}^{-2}\text{]}}$$

**Problem 5.** If I have a lamp from the hardware store whose efficiency is 50% and is drawing 2 amps at 60 volts, what will the flux on my radiometry notes be 2 meters away (assuming 8.5 in by 11 in paper)?

**Solution:**

The relation for efficiency is used to compute the light flux from the electrical flux.

$$\begin{aligned} \varepsilon &\triangleq \frac{\Phi_{out}}{P_{in}} \\ \Phi_{out} &= P_{in} \varepsilon = i v \varepsilon \\ E &\triangleq \frac{\partial \Phi_{out}}{\partial A_{sphere}} \\ \Phi_{paper} &= \frac{\partial \Phi_{out}}{\partial A_{sphere}} A_{paper} \\ \Phi_{paper} &= \frac{i v \varepsilon}{4\pi r^2} (A_{paper}) \\ \Phi_{paper} &= \frac{(2 \text{ A})(60 \text{ V})(0.50)}{4\pi(2 \text{ m})^2} (0.0603 \text{ m}^2) = \boxed{72 \text{ mW}} \end{aligned}$$

**Problem 6.** Given a source with 30% efficiency, drawing 0.2 amps from a 9 volt source, how far away from the source will the observer measure an irradiance of

$0.02 \text{ Wm}^{-2}$ ?

**Solution:**

The relation for efficiency is used to compute the light flux from the electrical flux.

$$\begin{aligned}\varepsilon &\triangleq \frac{\Phi_{out}}{P_{in}} \\ \Phi_{out} &= P_{in} \varepsilon = i v \varepsilon \\ E &\triangleq \frac{\partial \Phi_{out}}{\partial A} = \frac{\partial \Phi_{out}}{4\pi r^2} \\ r &= \left( \frac{\Phi_{out}}{4\pi E} \right)^{1/2} = \left( \frac{i v \varepsilon}{4\pi E} \right)^{1/2} \\ r &= \left( \frac{(0.2A)(9V)(0.30)}{4\pi(0.02 \text{ Wm}^{-2})} \right)^{1/2} = \boxed{1.46 \text{ m}}\end{aligned}$$

**Problem 7.** A sensor has a responsivity of 2 amps per watt, and an observed noise level of 100 milliamps. If the irradiance on a  $1 \text{ cm}^2$  detector changes by  $50 \text{ Wm}^{-2}$ , will the sensor record the change?

**Solution:**

We need to find out what is the signal (in amps) produced by the change in irradiance. Then we compare this to the noise value stated to see which is larger.

$$\begin{aligned}\beta &\triangleq \frac{\partial S}{\partial \Phi} \\ \partial S &= \beta \partial \Phi\end{aligned}$$

need to solve for the flux

$$\begin{aligned}E &\triangleq \frac{\partial \Phi}{\partial A} \\ \partial \Phi &= E \partial A\end{aligned}$$

after substitution and integrating out the partials we get

$$S = \beta E A = (2 \text{ AW}^{-1})(50 \text{ Wm}^{-2})(1 \times 10^{-4} \text{ m}^2) = \boxed{10 \text{ mA}}$$

Turns out, the noise is larger. The sensor will not record the change.

**Problem 8.** If you have a sensor with a responsivity of  $2 \text{ VW}^{-1}$ , an output of 5 volts, is  $1 \text{ cm}$  on a side, and is  $10 \text{ cm}$  from a source, determine the radiant intensity of the source.

**Solution:**

$$\begin{aligned}\beta &\triangleq \frac{\partial S}{\partial \Phi} \\ \partial \Phi &= \frac{\partial S}{\beta} \\ I &\triangleq \frac{\partial \Phi}{\partial \Omega} \\ I &= \frac{\partial S \backslash \beta}{\partial A \backslash r^2} = \frac{r^2 \partial S}{\partial A \beta} = \frac{(10 \text{ cm})^2 (5 \text{ V})}{(1 \text{ cm}^2)(2 \text{ VW}^{-1})} = \boxed{250 \text{ [Wsr}^{-1}\text{]}}\end{aligned}$$

**Problem 9.** If you have a source at a temperature of  $1000 \text{ }^\circ\text{K}$ , and an emissivity of 0.93, what is the radiant exitance of the source?

**Solution:**

$$\begin{aligned}\varepsilon &\triangleq \frac{M(T)}{M_{bb}(T)} \\ M(T) &= M_{bb}(T) \varepsilon = \sigma T^4 \varepsilon \\ M &= (5.6696 \times 10^{-8} \text{ [Wm}^{-2}\text{K}^{-4}\text{]})(1000 \text{ K})^4 (0.93) \\ &= \boxed{52,727 \text{ [Wm}^{-2}\text{]}}\end{aligned}$$

**Problem 10.** A satellite contains a detector with a responsivity of  $0.02 \text{ [AW}^{-1}\text{]}$ . As the detector spins, it is irradiated in three orientations (see Figure 1). What is the output of the cell in each of the three orientations if the source is located as shown in Figure 1 and has a radiance of  $0.3 \text{ [Wm}^{-2}\text{sr}^{-1}\text{]}$ ? Additionally, the source has an area of  $4 \text{ cm}^2$  while the detector has an area of  $1 \text{ cm}^2$ .

**Solution:**

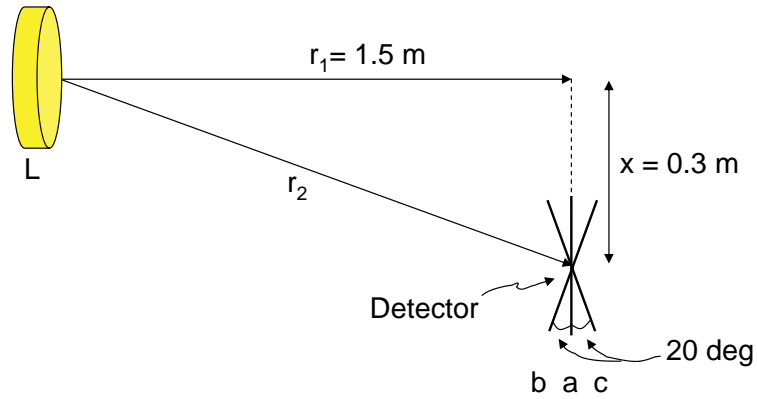


Figure 1: Layout for satellite detector problem.

We can use the expression relating radiance and intensity. That is,

$$L = \frac{I_o}{A \cos \theta}$$

$$I_o = LA \cos \theta$$

If we consider the radiance on-axis (straight out) then  $\cos \theta$  is one. Therefore we have

$$I_o = LA$$

If we assume the source is Lambertian, then the intensity varies as

$$I_\theta = I_o \cos \theta$$

and since we wish to point the intensity in the direction of the detector we have

$$I_\theta = LA \cos \theta$$

Now we would like to calculate the irradiance onto the detector. Can we use the inverse square law for point sources? Is the distance to the detector at least 10 times the diameter of the source? The distance to the detector is

$$r_2 = \sqrt{r_1^2 + x^2} = \sqrt{(1.5 \text{ m})^2 + (0.3 \text{ m})^2} = 1.53 \text{ m}$$

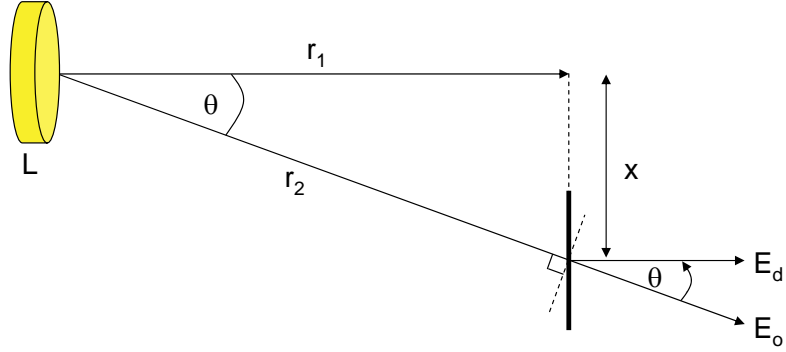


Figure 2: Layout for satellite detector problem, orientation (a).

The diameter of the source is

$$A_s = \frac{\pi d_s^2}{4}$$

$$d_s = \sqrt{\frac{4A_s}{\pi}} = \sqrt{\frac{(4)(4 \text{ cm}^2)}{\pi}} = 2.25 \text{ cm}$$

Ten times this number is  $20.25 \text{ cm} = 0.2 \text{ m}$  which is much less than our distance of  $1.53 \text{ m}$ . Using the inverse square law to find the irradiance onto the detector we have (as in Figure 2)

$$E_o = \frac{I_\theta}{r_2^2}$$

$$= \frac{L A_s \cos \theta}{r_2^2}$$

We now need to take into account projected area effects to obtain the irradiance,  $E_d$  onto the detector.

$$E_d = E_o \cos \theta = \frac{L A_s \cos^2 \theta}{r_2^2}$$

We would like to compute the flux,  $\Phi_d$  onto the detector.

$$E_d \triangleq \frac{\partial \Phi_d}{\partial A_d}$$

$$\partial \Phi_d = E_d \partial A_d = \frac{L A_s \partial A_d \cos^2 \theta}{r_2^2}$$

At this point we can use the definition for responsivity and re-write in terms of the signal,  $S_d$ .

$$\beta_d \triangleq \frac{\partial S_d}{\partial \Phi_d}$$

$$\partial S_d = \beta_d \partial \Phi_d = \frac{\beta_d L A_s \partial A_d \cos^2 \theta}{r_2^2}$$

After integrating out the partial derivatives, we have

$$S_d = \frac{\beta L A_s A_d \cos^2 \theta}{r_2^2}$$

We still need to calculate the angle,  $\theta$ . Using trigonometry, we have

$$\tan \theta \triangleq \frac{x}{r_1}$$

$$\theta = \arctan\left(\frac{x}{r_1}\right) = \arctan\left(\frac{0.3m}{1.5m}\right) = 11.3 \text{ deg}$$

Final substitution yields

$$S_d = \frac{\beta L A_s A_d \cos^2 \left[ \arctan\left(\frac{x}{r_1}\right) \right]}{\left(\sqrt{r_1^2 + x^2}\right)^2}$$

$$= \frac{(0.02 [AW^{-1}])(0.3 [Wm^{-2}sr^{-1}])(4 \times 10^{-4}m^2)(1 \times 10^{-4}m^2) \cos^2(11.3 \text{ deg})}{(1.53 m)^2}$$

$$= 9.86 \times 10^{-11} A = \boxed{98.6 \text{ pA}} \text{ for position (a).}$$

**Problem 11.** Assuming the Sun is approximately a 5700 °K blackbody radiator with a diameter of 1,392,000 kilometers, what is a) the wavelength of maximum spectral radiant exitance? b) the spectral radiant exitance at this wavelength? c) the total radiant exitance of the Sun? d) the total power output of the Sun?

**Solution:**

a) The wavelength of maximum spectral radiant exitance?

$$\lambda_{peek} \triangleq \frac{A}{T} = \frac{2898 \mu m K}{5700 K} = \boxed{0.508 \mu m}$$

b) The spectral radiant exitance at this wavelength?

$$\begin{aligned}
 M(\lambda, T) &\triangleq \frac{2\pi hc^2}{\lambda^5 (\exp(\frac{hc}{\lambda kT}) - 1)} \\
 M(0.508 \mu m, 5700 K) &= \frac{2\pi (6.6 \times 10^{-34} J s) (3 \times 10^8 m s^{-1})^2}{(0.508 \times 10^{-6} m)^5 (\exp(\frac{(6.6 \times 10^{-34} J s)(3 \times 10^8 m s^{-1})}{(0.508 \times 10^{-6} m)(1.38 \times 10^{-23} J K^{-1})(5700 K)}) - 1)} \\
 &= \frac{3.732 \times 10^{-16} J m^2 s^{-1}}{(0.508 \times 10^{-6} m)^5 (\exp(\frac{1.98 \times 10^{-25} J m}{3.99 \times 10^{-26} J m}) - 1)} \\
 &= \frac{3.732 \times 10^{-16} J m^2 s^{-1}}{(0.508 \times 10^{-6} m)^5 (140.89)} \\
 &= \frac{3.732 \times 10^{-16} J m^2 s^{-1}}{4.766 \times 10^{-30} m^5} \\
 &= 7.83 \times 10^{13} J s^{-1} m^{-3} \\
 &= \boxed{7.83 \times 10^7 [W m^{-2} \mu m^{-1}]}
 \end{aligned}$$

c) The total radiant exitance of the Sun?

$$M_{total} \triangleq \sigma T^4 = (5.67 \times 10^{-8} W m^{-2} K^{-4})(5700 K)^4 = \boxed{5.98 \times 10^7 [W m^{-2}]}$$

d) The total power output of the Sun?

$$\begin{aligned}
 \Phi_{total} &= M_{total} A_{sun} = M_{total} (4\pi r_{sun}^2) \\
 &= (5.98 \times 10^7 W m^{-2}) \left( 4\pi \left( \frac{1.392 \times 10^9 m}{2} \right)^2 \right) \\
 &= \boxed{3.64 \times 10^{26} W}
 \end{aligned}$$

**Problem 12.** Given that the diffuser shown in Figure 3 produces a flux  $\Phi_\theta$  through the aperture on the right hand side and that the diffuser's intensity is observed to vary according to  $I_\theta = I_o \cos^2 \theta$ , derive completely, in simplest form, an expression for the intensity,  $I_o$  in terms of  $\Phi_\theta$ . State all assumptions. (Hint:  $\int \cos^m x \sin x \partial x = \frac{-\cos^{m+1} x}{m+1}$ ).

**Solution:**

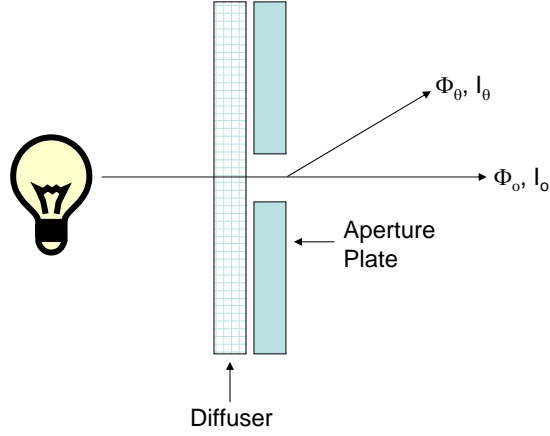


Figure 3: Illustration of flux onto a diffuser, followed by flux exiting through an aperture.

If we assume that the aperture is small compared to the measured distance, we can use point source approximations. We can first state the on-axis ( $E_o, I_o$ ) case which leads to,

$$\begin{aligned}
 E_o &= \frac{I_o}{r^2} \\
 \frac{\partial \Phi_o}{\partial A} &= \frac{I_o}{r^2} \\
 \partial \Phi_o &= \frac{I_o \partial A}{r^2}
 \end{aligned}$$

The off-axis case is then

$$\begin{aligned}
 \partial \Phi_\theta &= \frac{I_\theta \partial A}{r^2} \\
 &= \frac{I_o \cos^2 \theta (r \, d\theta \cdot r \sin \theta \, d\phi)}{r^2} \\
 &= I_o \cos^2 \theta \sin \theta \, d\theta \, d\phi
 \end{aligned}$$

Integrating both sides we have

$$\begin{aligned}
 \Phi_\theta &= I_o \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \cos^2 \theta \sin \theta \, d\theta \, d\phi \\
 &= I_o \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi/2} \cos^2 \theta \sin \theta \, d\theta \\
 &= 2\pi I_o \left( \left. \frac{-\cos^3 \theta}{3} \right|_0^{2\pi} \right) \\
 &= \frac{2\pi I_o}{3} \\
 I_o &= \boxed{\frac{3\Phi_\theta}{2\pi}}
 \end{aligned}$$

**Problem 13.** Given a lamp with a radiant intensity of  $0.1 \text{ [Wsr}^{-1}\text{]}$  illuminating a lambertian diffuser  $10 \text{ cm}$  away with a  $1 \text{ cm}$  diameter aperture located just beyond the diffuser; this “new”  $1 \text{ cm}$  diameter source then illuminates a detector  $100 \text{ cm}$  from the lamp (see Figure 4). What is the irradiance on the detector if the transmission of the diffuser is  $0.60$ ?

**Solution:**

We will assume the radius of the lamp is small relative to the distance,  $r_1$ . The irradiance  $E_i$  incident onto the back of the diffuser at a distance  $r_1$  is.

$$E = \frac{I}{r_1^2}$$

The exitance  $M$  and flux  $\Phi$  from the diffuser is

$$\begin{aligned}
 M &= E\tau = \frac{I\tau}{r_1^2} \\
 \Phi &= MA = \frac{I\tau A}{r_1^2}
 \end{aligned}$$

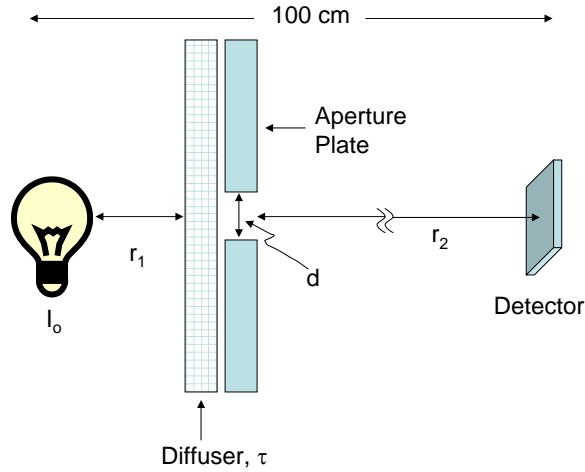


Figure 4: Illustration of flux onto a diffuser from a source, followed by flux exiting through an aperture or diameter,  $d$  which is incident on a detector at a distance,  $r_2$  away.

Since the surface is lambertian, we can use a version of the relation  $M = L\pi$ . For the on-axis case from the diffuser we have

$$\begin{aligned}
 I_{diff} &= \frac{\Phi}{\pi} \\
 &= \frac{I\tau A}{\pi r_1^2}
 \end{aligned}$$

Assuming the “new” source,  $I_{diff}$  acts like a point-source (e.g., distance is at least 10 times the diameter of the source), we can use the

inverse square law. Thus we have

$$\begin{aligned}
 E_d &= \frac{I_{diff}}{r_2^2} \\
 &= \frac{I \tau A}{\pi r_1^2 r_2^2} \\
 &= \frac{I \tau \pi d^2}{4\pi r_1^2 r_2^2} \\
 E_d &= \boxed{\frac{I \tau d^2}{4 r_1^2 r_2^2}} \\
 &= \frac{(0.1 [Wsr^{-1}]) (0.6)(0.01 m)^2}{4(0.1 m)^2(0.9 m)^2} \\
 &= 1.85 \times 10^{-4} [Wm^{-2}] = \boxed{185 [\mu Wm^{-2}]}
 \end{aligned}$$

### PMT Problems

**Problem 14.** Given a PMT with a cathode responsivity of 0.05 amps per watt, cathode size of 0.31 x 0.98 inches, anode sensitivity of 100,000 [ $A cm^2 W^{-1}$ ], maximum anode current of 0.005 amps, what are the a) anode responsivity, b) gain, c) maximum cathode current, d) maximum cathode flux and, e) the maximum irradiance?

**Solution:**

a) Anode responsivity?

$$\begin{aligned}
 \beta_a &\triangleq \frac{i_a}{\Phi_c} \\
 &= \frac{i_a}{E_c A_c} = \frac{\beta'_a}{A_c} = \frac{\beta'_a}{lw} \\
 &= \frac{100,000 [A cm^2 W^{-1}]}{(0.31 in)(0.98 in) \left(\frac{2.54cm}{1in}\right)^2} \\
 &= \boxed{51,020 [A W^{-1}]}
 \end{aligned}$$

b) Gain?

$$\begin{aligned} G &\triangleq \frac{i_a}{i_c} \\ &= \frac{\beta_a \Phi_c}{\beta_c \Phi_c} = \frac{\beta_a}{\beta_c} \\ &= \frac{51,020 [A W^{-1}]}{0.05 [A W^{-1}]} \\ &= \boxed{1.02 \times 10^6} \end{aligned}$$

c) Maximum cathode current?

$$\begin{aligned} G &\triangleq \frac{i_a}{i_c} = \frac{i_{a,max}}{i_{c,max}} \\ i_{c,max} &= \frac{i_{a,max}}{G} \\ &= \frac{0.005 A}{1.02 \times 10^6} \\ &= 4.9 \times 10^{-9} A = \boxed{4.9 nA} \end{aligned}$$

d) Maximum cathode flux?

$$\begin{aligned} \beta_c &\triangleq \frac{i_c}{\Phi_c} \\ \Phi_{c,max} &= \frac{i_{c,max}}{\beta_c} \\ &= \frac{4.9 nA}{0.05 [A W^{-1}]} \\ &= 9.8 \times 10^{-8} W = \boxed{98 nW} \end{aligned}$$

e) Maximum irradiance?

$$\begin{aligned} E &\triangleq \frac{\partial \Phi}{\partial A} \\ E_{c,max} &= \frac{\Phi_{c,max}}{A_c} \\ &= \frac{98 \times 10^{-9} W}{(0.31 \text{ in})(0.98 \text{ in}) \left(\frac{0.0254 m}{1 \text{ in}}\right)^2} \\ &= 5 \times 10^{-4} [W m^{-2}] = \boxed{500 [\mu W m^{-2}]} \end{aligned}$$

**Problem 15.** For the previous PMT above, given an incident irradiance of  $1.0 [\mu W m^{-2}]$ , what is the PMT current?

**Solution:**

$$\begin{aligned} G &\triangleq \frac{i_a}{i_c} \\ i_a &= G i_c = G(\beta_c \Phi_c) = G \beta_c E_c A_c \\ &= (1.02 \times 10^6) (0.05 [AW^{-1}]) (1.0 \times 10^{-6} [Wm^{-2}]) \left( (0.31 \text{ in})(0.98 \text{ in}) \left( \frac{0.0254 \text{ m}}{1 \text{ in}} \right)^2 \right) \\ &= 10 \times 10^{-6} A = \boxed{10 \mu A} \end{aligned}$$