Control of Light

Emmett Ientilucci
Digital Imaging and Remote Sensing Laboratory
Chester F. Carlson Center for Imaging Science
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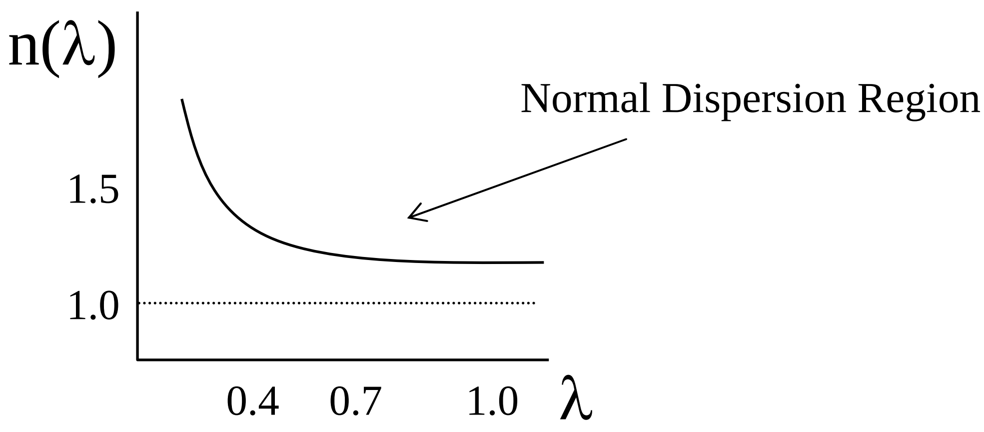
Spectro-radiometry

- Spectral Considerations
  - Chromatic dispersion
  - Diffraction grating theory
- Spectrometer
  - Filter Spectrometer
  - Prism Spectrometer
  - Grating Spectrometer
  - Applications and numerical example
- Diffraction Grating Types
  - Plane Grating
  - Concave Grating
- Wavelength Selectors
  - Filters
    - Interference Filters
    - Interference Wedges
    - Absorption Filters
  - Monochromators

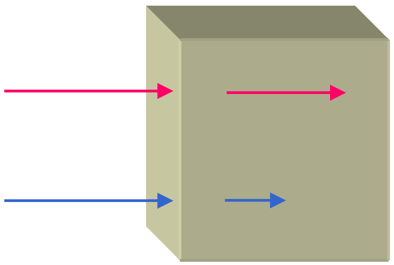
Spectral Considerations

$$n(\lambda) = \frac{\text{speed in vacuum}}{\text{speed in medium}} = \frac{c}{v(\lambda)}$$

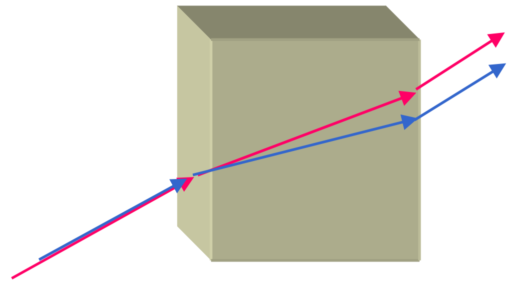
-n, will always be greater than one
-So, the index is different for different wavelengths
-Therefore, the amount of refraction is different
-This effect is called chromatic dispersion



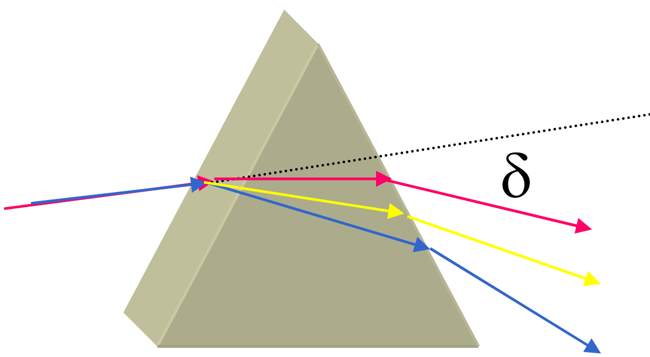
Spectral Considerations



Blue Lags Red



Oblique angle, colors separate
(Snells Law)



Dispersing Prism
δ is the deviation angel

Applications of Dispersing Prisms

- Spectro-*scope*
  - Used for view a spectrum
- Specto-*meter*
  - Is equipped for measuring a spectrum
- Specto-*graph*
  - Is built for photography
- Specto-*photometer*
  - Photocell takes the place of the photographic film
- Mono-chromator
  - Instrument for selecting light of different wavelengths

Wave Optics

- Interference
  - Superposition of waves
  - Young’s double-slit experiment
  - Interferometers
  - Coherence: spatial/temporal/partial
- Diffraction
  - Fraunhofer diffraction
  - Fresnel diffraction
- Polarization

Wave Optics

- Diffraction Gratings
  - N-slit interference (diffraction grating)
  - Grating equation
  - Resolution
  - Types of gratings
- Thin Films
  - Plane-parallel plates
  - Faby-Perot interferometer
  - Newton's rings
  - Interference filters
  - Antireflection coatings

Interference

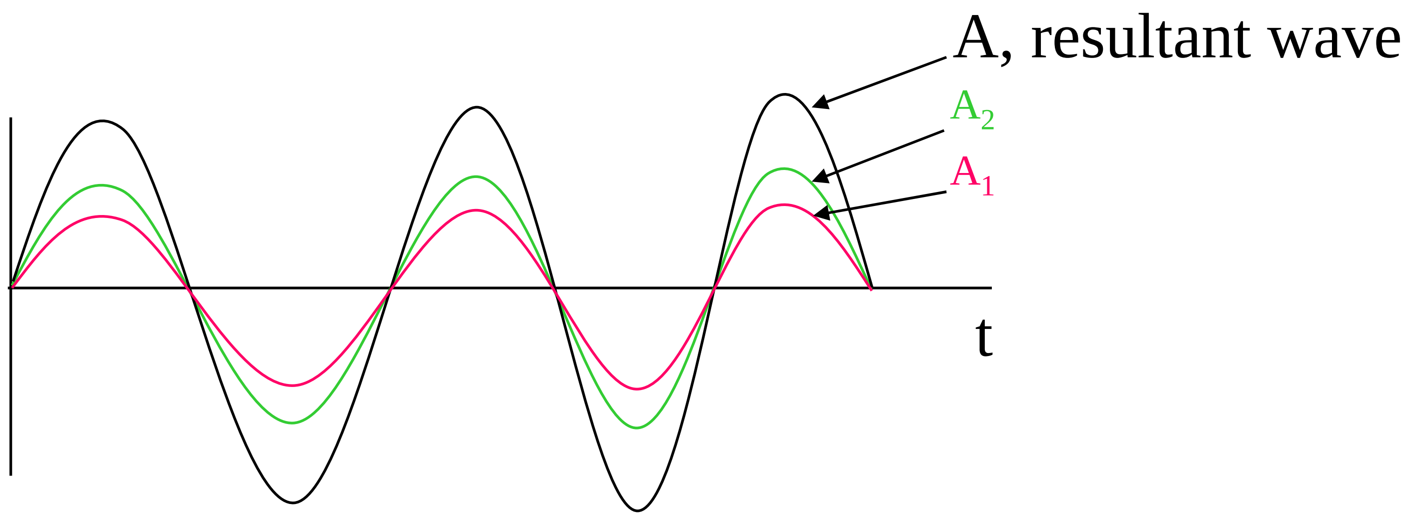
- Refers to the phenomenon that waves, under certain conditions, intensify or weaken each other.

- Interference is inseparably tied to that of *diffraction (as we will see)*.

Superposition of Waves

- Consider the following 2 cases
- The superposition of waves of
  1. Equal phase and frequency
  2. Constant phase difference

Equal phase and frequency



$$A = A_1 + A_2 + \ldots + A_N$$

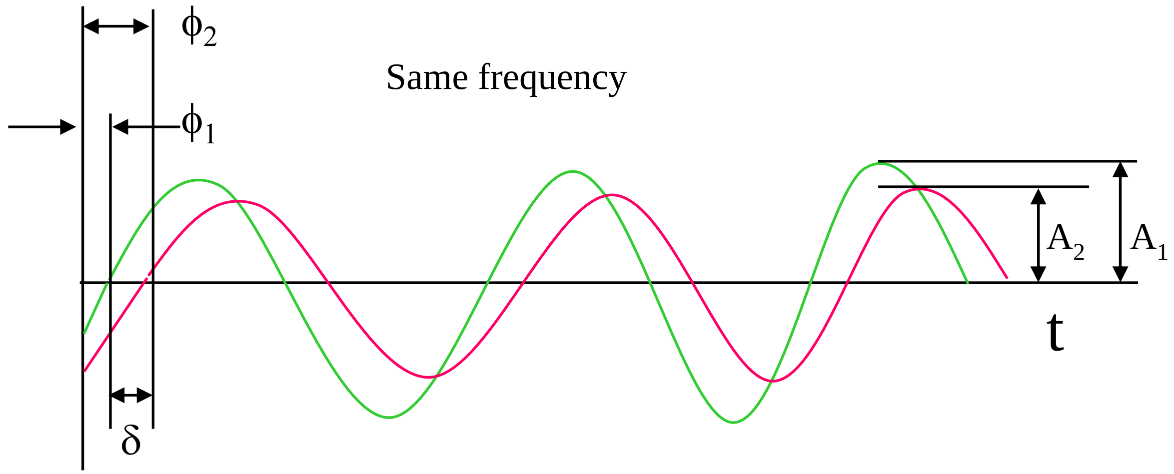
Irradiance of light is proportional to the square of the amplitude

$$Irradiance \propto A^2$$

$$E \propto A^2$$

$$E \propto \left(A_1 + A_2 + A_3 + \cdots + A_N\right)^2$$

Constant phase difference



The resultant wave can be found by using complex algebra

$$\mathbf{A}_1 = A_1 e^{i(\omega t+\phi_1)} \longrightarrow RE = A_1 \cos(\omega t + \phi_1)$$

$$\mathbf{A}_2 = A_2 e^{i(\omega t+\phi_2)}$$

$$\mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2$$

$$\mathbf{A}^2 = \mathbf{A}_1{}^2 + \mathbf{A}_2{}^2 + 2\mathbf{A}_1\mathbf{A}_2 \cos(\phi_1 - \phi_2)$$

We want irradiance, i.e., A^2

Interference Fringes

$$\mathbf{A}^2 = \mathbf{A}_1{}^2 + \mathbf{A}_2{}^2 + 2\mathbf{A}_1\mathbf{A}_2 \cos(\phi_1 - \phi_2)$$

$$\mathbf{E} \propto \mathbf{A}^2$$

$$\mathbf{E} = E_1 + E_2 + 2\sqrt{E_1 E_2}\cos\delta$$

We can rewrite the irradiance with substitution

When $\delta = 0$ (in phase), get maximum amount of light

$$\mathbf{E}_{\mathbf{max}} = E_1 + E_2 + 2\sqrt{E_1 E_2}$$

Furthermore, if $E_1 = E_2$ (e.g., coming from a LASER source)

$$\mathbf{E}_{\mathbf{max}} = 4E_1$$

(constructive interference)

Interference Fringes

When $\delta = 180$ (out of phase), get minumun amount of light

$\mathbf{E}_{\mathbf{min}} = E_1 + E_2 - 2\sqrt{E_1 E_2}$

If $E_1 = E_2$

$\mathbf{E}_{\mathbf{min}} = 0$ (destructive interference)

In general, assuming $E_1 = E_2 = E_0$

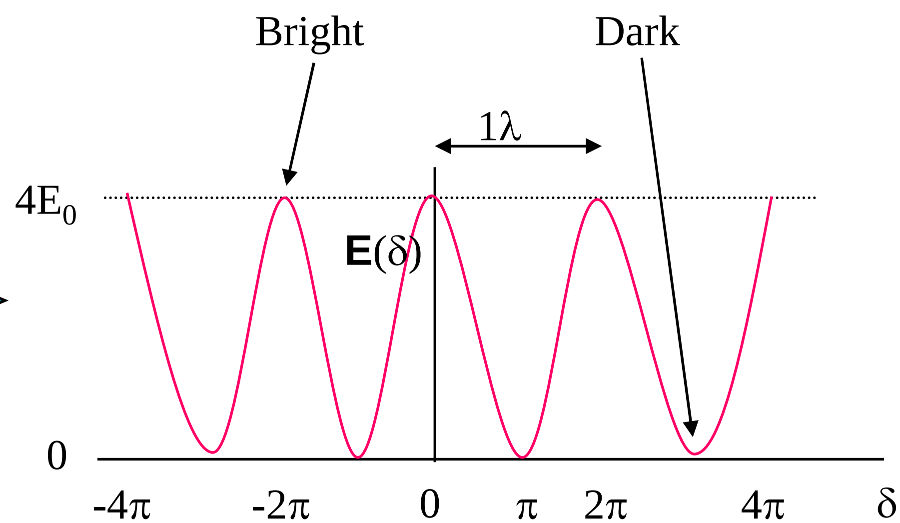
$\mathbf{E} = E_0 + E_0 + 2E_0 \cos\delta$

$\mathbf{E} = 2E_0(1 + \cos\delta)$

$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

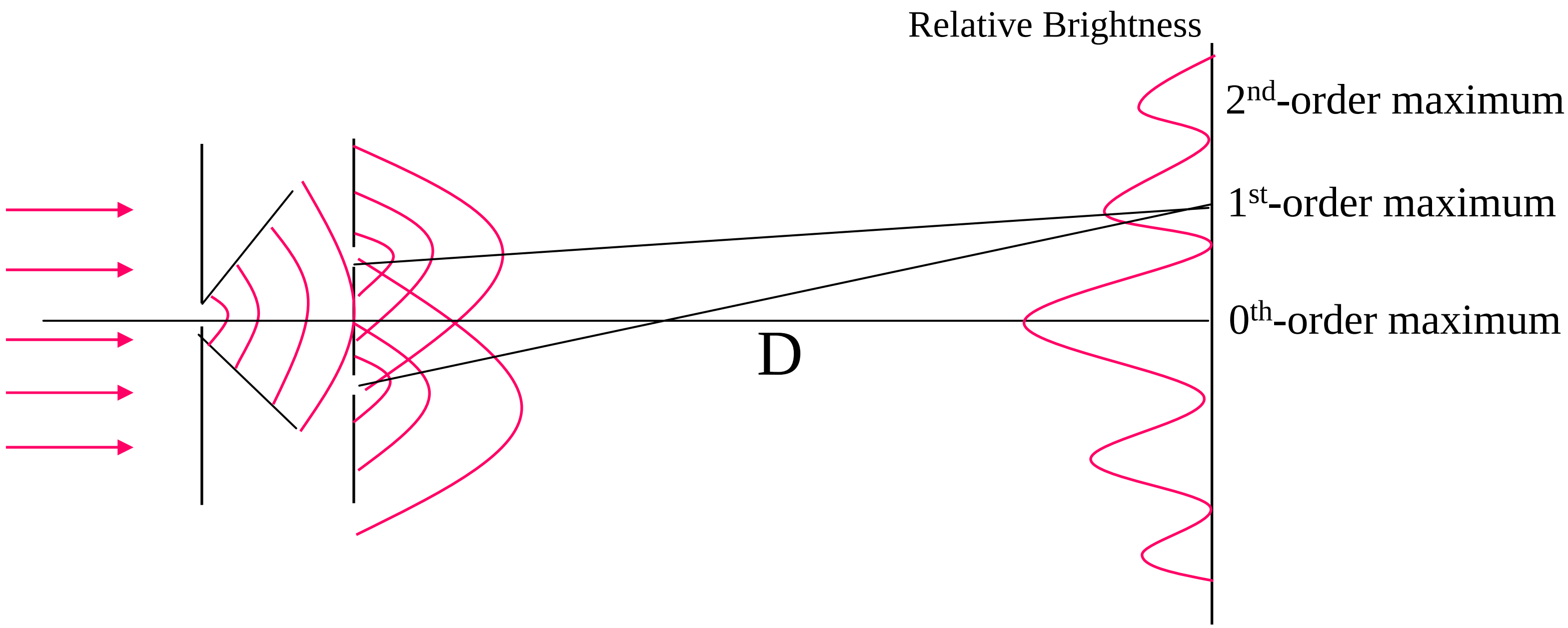
$\mathbf{E} = 4E_0 \cos^2(\frac{\delta}{2})$

Alternating bright/dark fringes called interference



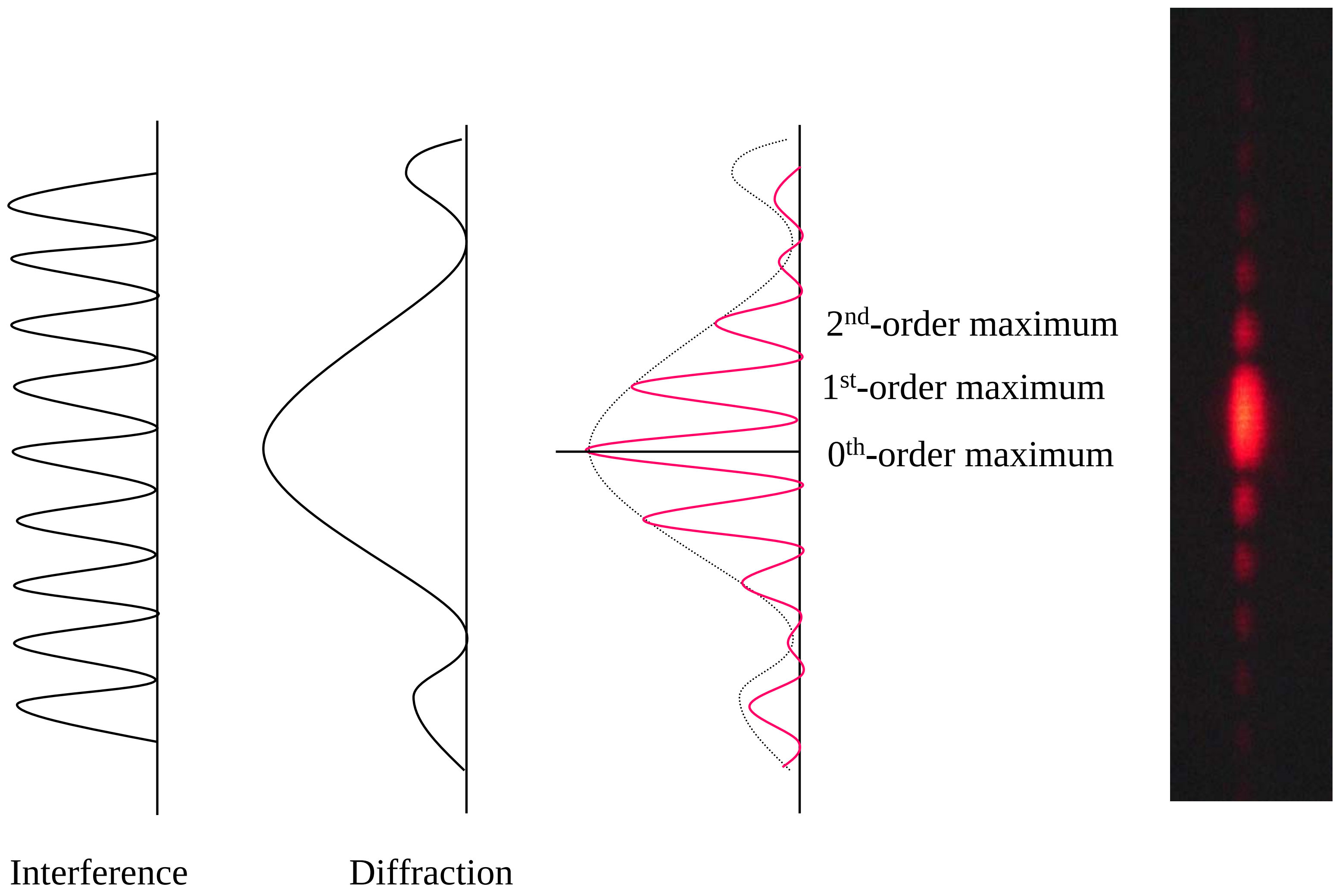
Young's Double-Slit Experiment

2-Beam Interference (monochromatic source)
Assume separation of slits is << D (i.e., Fraunhofer, or far field)

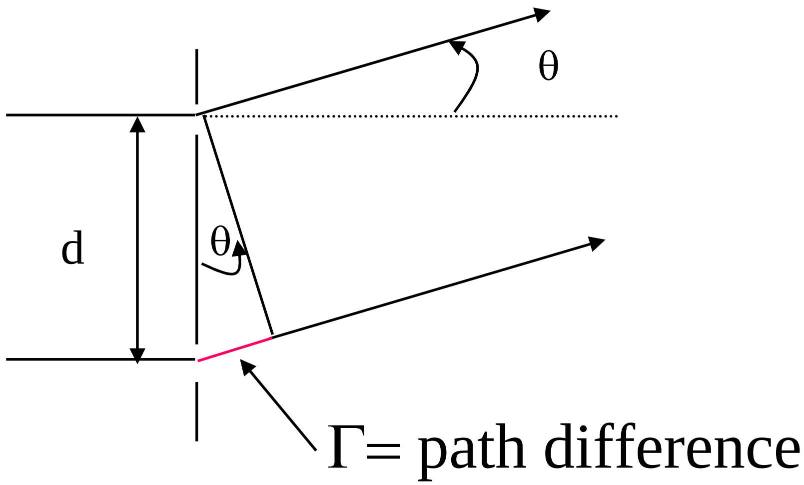


Slits widths are not much greater in width than the wavelength of light

Young’s Double-Slit Experiment



Young's Double-Slit Experiment



-When the path difference is zero, we have 0^{th} order maximum

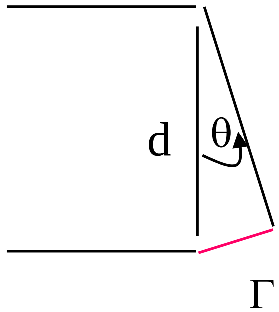
-But a maximum will occur whenever the path difference, Γ is <u>one wavelength</u> or an integral multiple of a wavelength, $m\lambda$ (remember the phase difference from before)

- Lets define the path difference as:

$$\Gamma = m\lambda$$

The integer *m* is called the *order of interference*

Young's Double-Slit Experiment



Position of the Maximum (from geometry)

$$\sin\theta = \frac{\Gamma}{d} = \frac{m\lambda}{d}$$

$$\boxed{d\sin\theta = m\lambda}$$

$m = 0,1,2...$

ϕ difference depends on angle

(We will see that this becomes the grating equation)

Positions of the Minimum

-whenever one of the contributions has shifted in phase by λ/2, that is

$$\boxed{d\sin\theta = (m - \tfrac{1}{2})\lambda}$$

$m = 1,2...$

In double-slit interference there is no zeroth-order minimum (i.e., m=0)

Diffraction

- When light passes through a narrow slit, it spreads out more than what could be accounted for by geometric construction

- Diffraction can be defined as any departure from *the predictions of geometric optics*

What is a Diffraction Grating?

- Extension of Young's double slit
- Is based on both diffraction *and* interference
- Uses an arrangement which is equivalent in its action to a number of parallel equidistant slits of the same width
- Move from 2-slit to N-slit interference problem

Intensity Distribution from and Ideal Grating

- So far we have considered only the separation, *d*, of the slits.
- But the slits also have a finite width, *s*.
- This significantly changes the intensity distribution behind the grating.

- This leads to a diffraction contribution due to the finite width, s of the slits, in addition to interference effects from the separation, d.

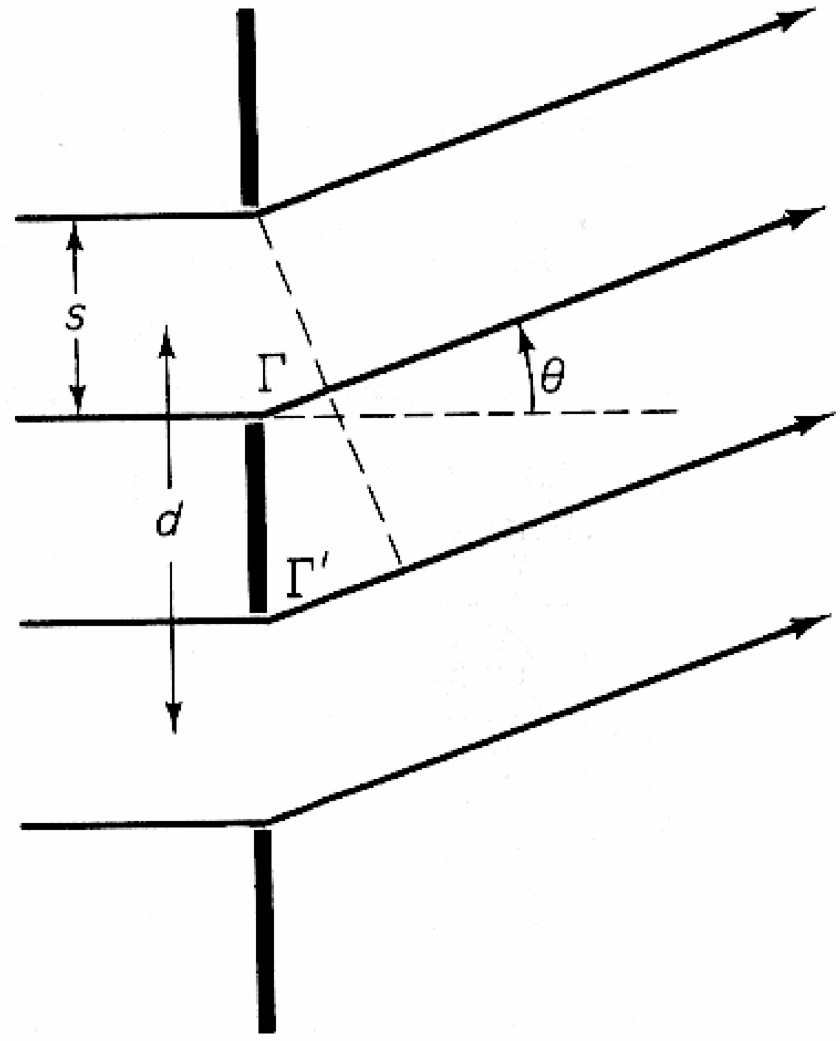
Ideal Grating

Eq. (1) $$E_\theta = E_0 \frac{\sin^2\left(\frac{\pi}{\lambda} s \sin\theta\right)}{\left(\frac{\pi}{\lambda} s \sin\theta\right)^2}$$

-Diffraction contribution of **single slit**.
-Due to finite width, *s*, of the slits.

Eq. (2) $$E_\theta = E_0 \frac{\sin^2\left(N\frac{\pi}{\lambda} d \sin\theta\right)}{\sin^2\left(\frac{\pi}{\lambda} d \sin\theta\right)}$$

-This is the interference contribution.
-Due to multiplicity of slits separated by *d*



Ideal Grating - Show Applet

The total, actual pattern behind the grating is found by **multiplying the two contributions**.

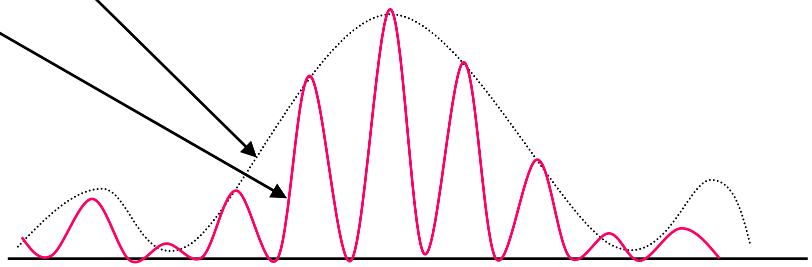
Eq. (3) $$E_\theta = E_0 \frac{\sin^2 D}{D^2} \frac{\sin^2 NI}{\sin^2 I}$$

-One contribution is being modulated by the other
-Diffraction is modulating the interference term

Where,

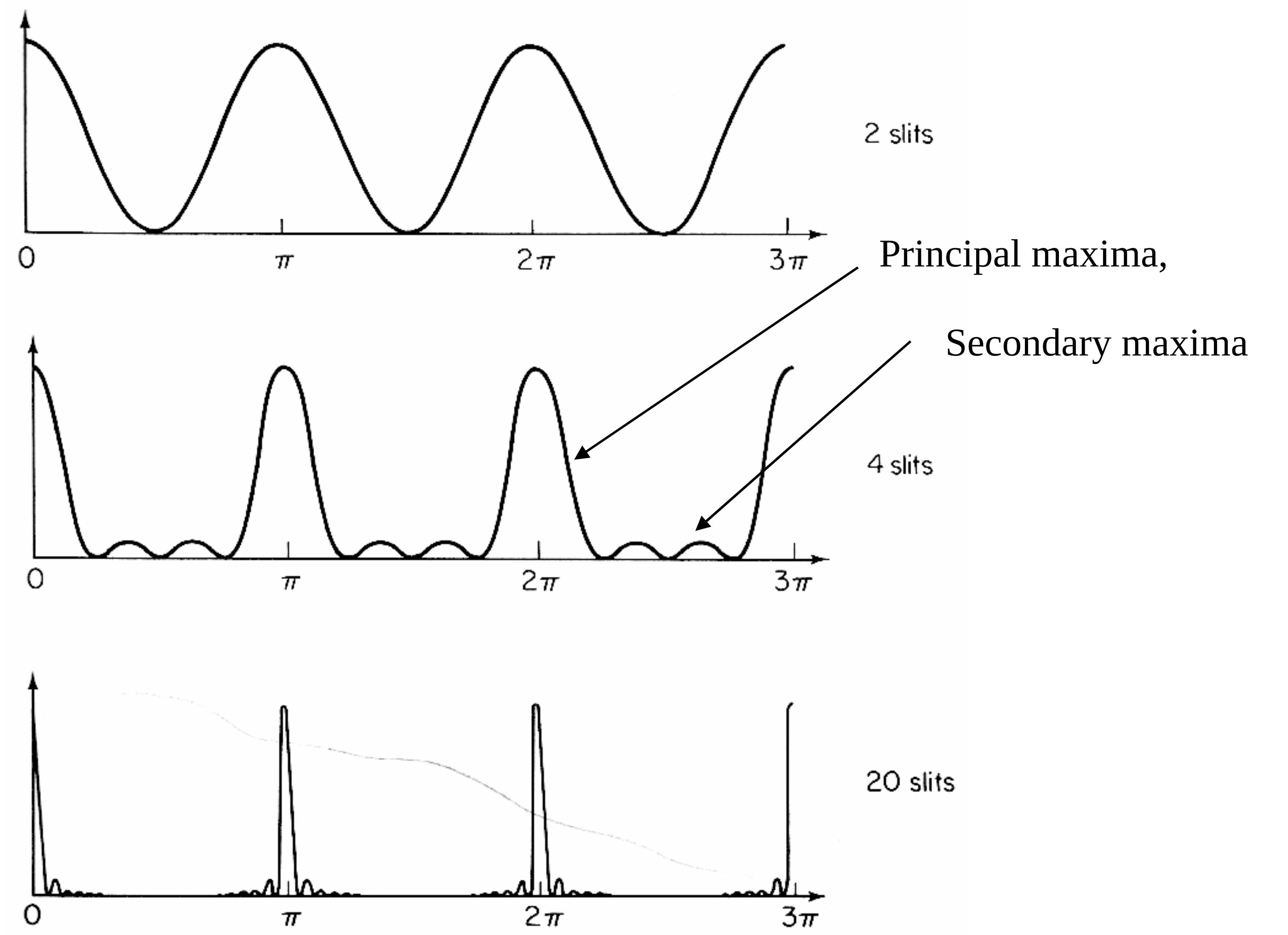
$$D = \frac{\pi b \sin\theta}{\lambda}$$ Pertinent term in the diffraction contribution

$$\mathrm{I} = \frac{\pi \mathrm{d} \sin\theta}{\lambda}$$ Pertinent term in the interference contribution

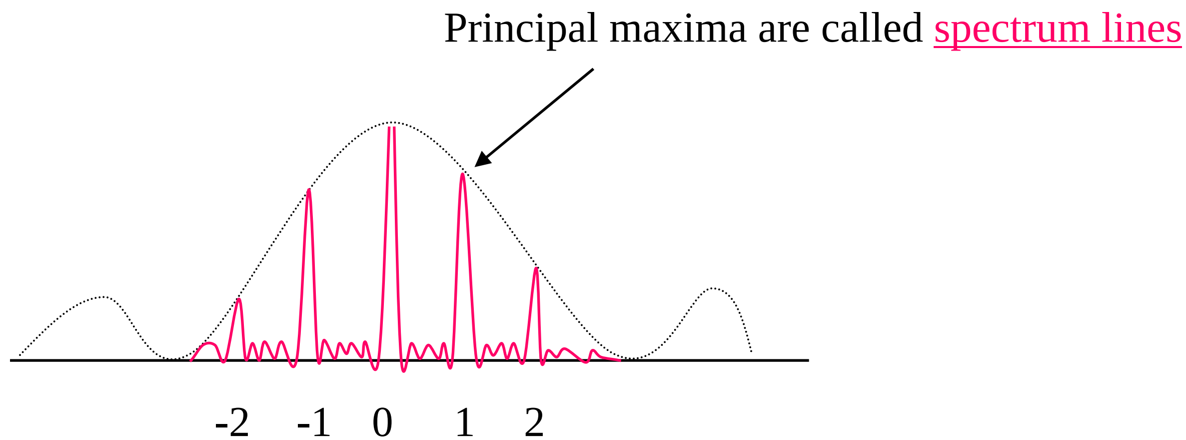


N-Slits

-When you increase number of slits, **N,** the **interference** maxima **narrows**
-At the same time, the secondary maxima, SM, between them are suppressed

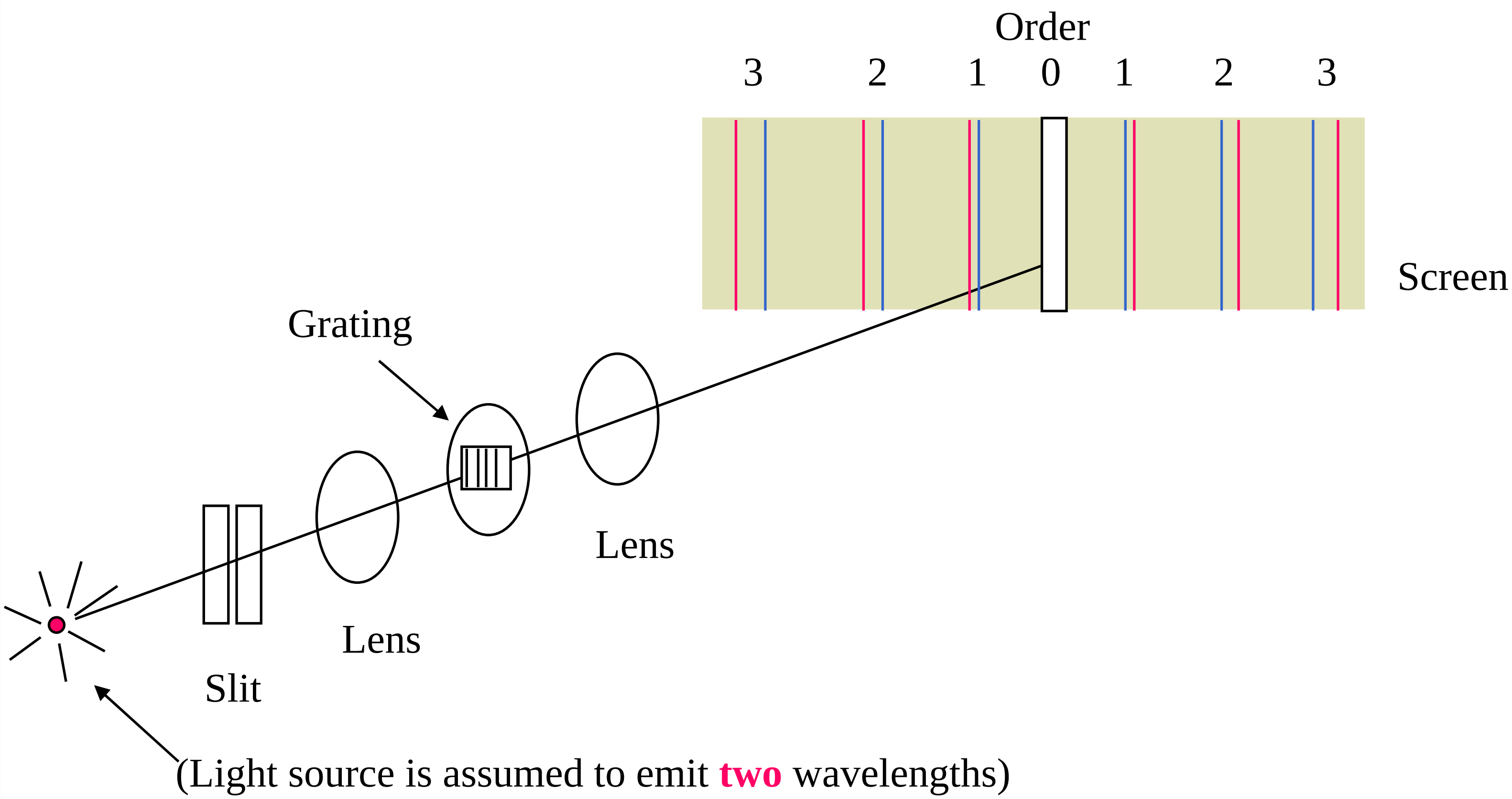


Formation of Spectra by Grating

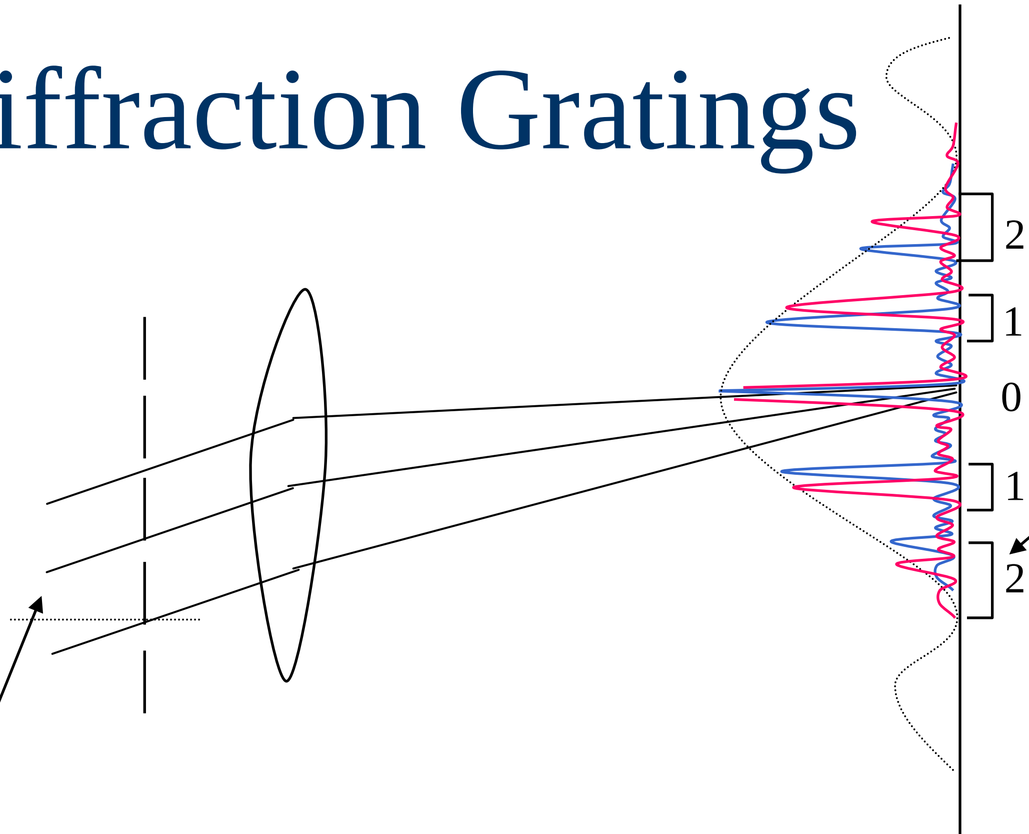


Transmission Diffraction Grating

Principal uses are in spectroscopy
e.g., Transmission Grating Spectrograph,
(similar to prism spectrograph)

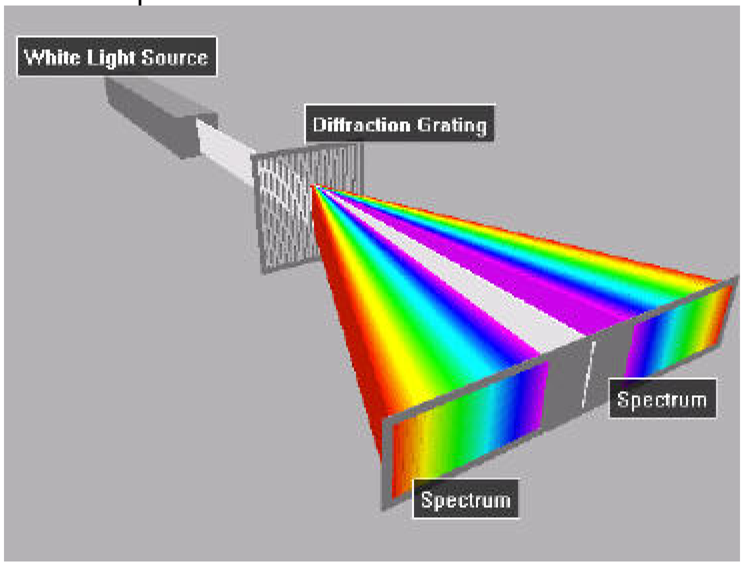


Diffraction Gratings



Actually would get a “**line spectrum**”

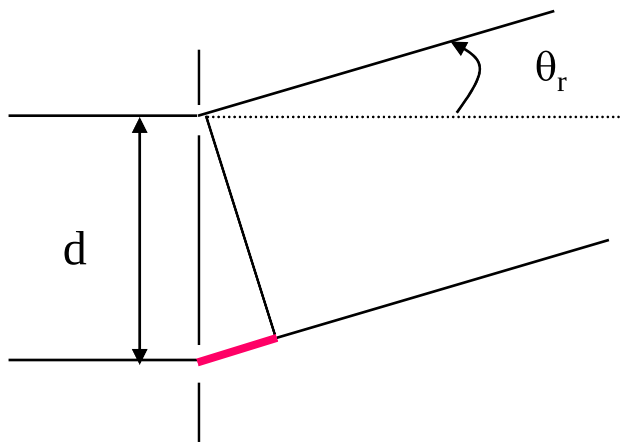
(Light source is assumed to emit **two** wavelengths)



Diffraction Gratings - Formally

As in double-slit interference from before:

d = the distance between the centers of *any* two adjacent slits
θ_r= the angle through which the light is diffracted
m = order
λ = wavelength



$$d \sin \theta_r = m\lambda$$

$m = 0,1,2...$

This is the same equation derived for double-slit maxima or the grating equation for ***normal incidence***
(*e.g.*, can compute the angles at which the PM are formed)

General Diffraction Grating Eqn.

If light is incident at an angle θ_i, then

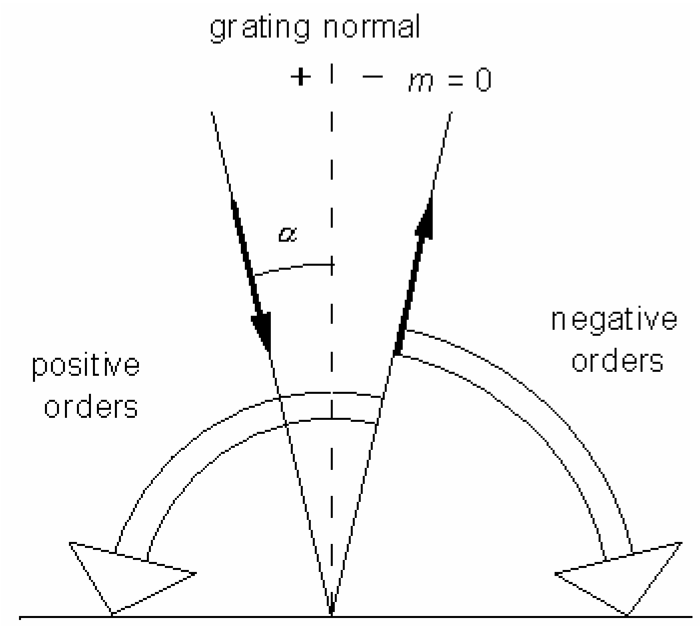
$$d(\sin\theta_i \pm \sin\theta_r) = m\lambda$$ Eq. (5)

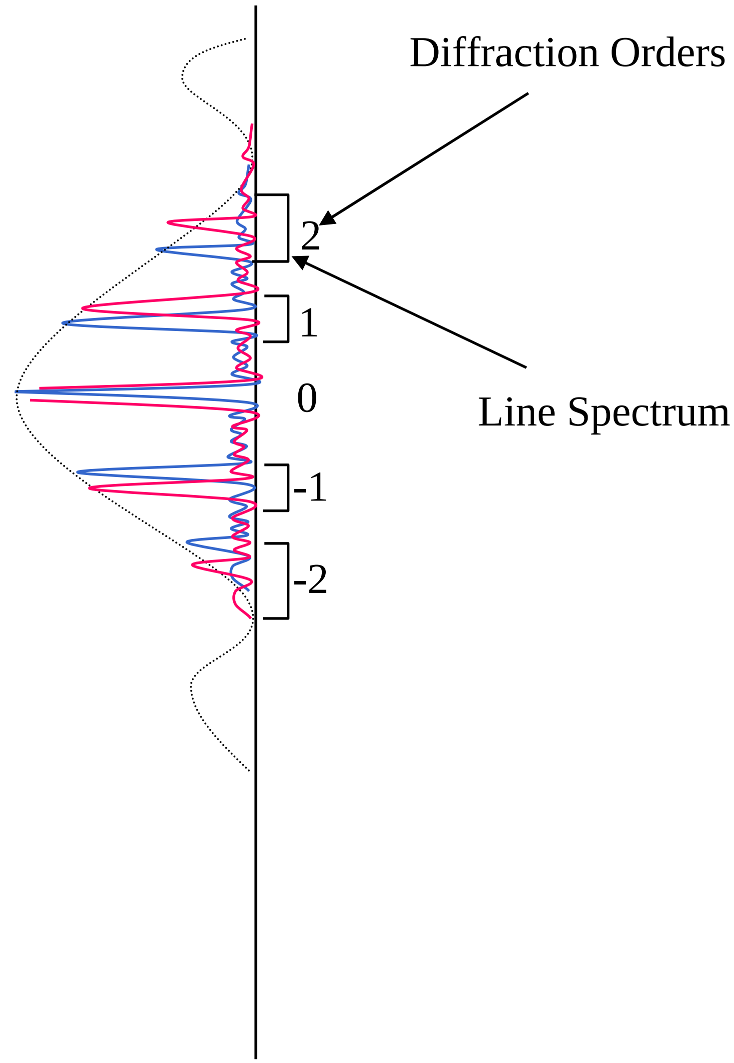
Which is the more complete grating equation

(-) indicates θ_r and θ_i are on opposite sides of the grating normal.

(+) indicates θ_r and θ_i are on same side of grating normal

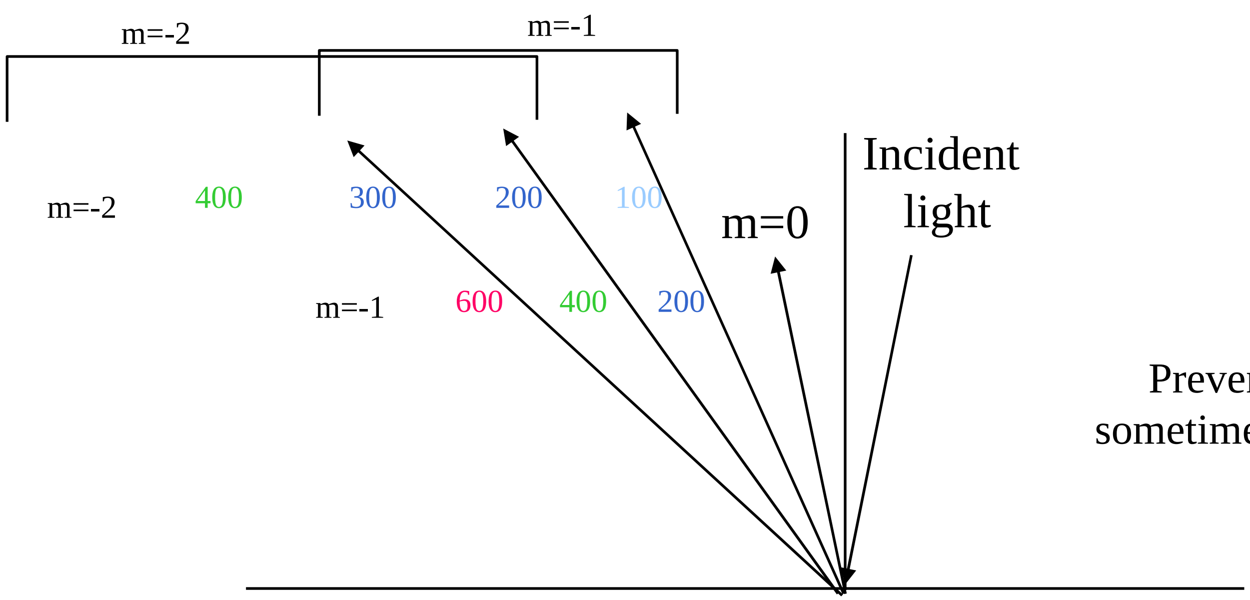
Diffraction Orders





Overlapping Diffracted Spectra

- Problem with multiple order behavior is that successive spectra overlap



Prevented by filtering or sometimes called order sorting

Light for λ=100, 200, 300 nm in the 2nd order is diffracted in the same direction as light λ = 200, 400, 600 nm in the 1st order.

6.9 Spectro-radiometry

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  - Chromatic dispersion
  - Diffraction grating theory
- Spectrometer
  - Filter Spectrometer
  - Prism Spectrometer
  - Grating Spectrometer
  - Applications and numerical example
- Diffraction Grating Types
  - Plane Grating
  - Concave Grating
- Wavelength Selectors
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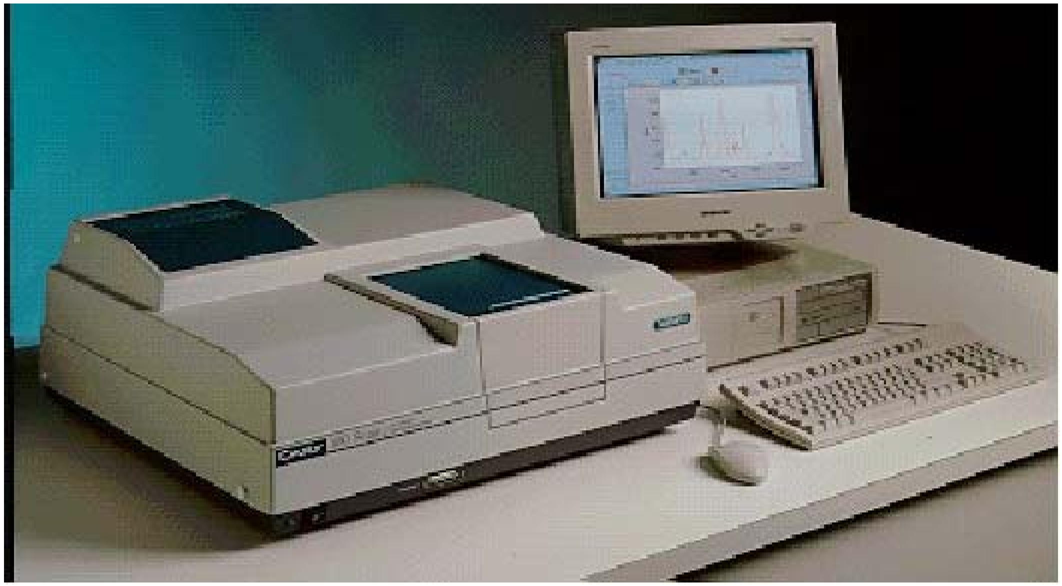
Spectrometer

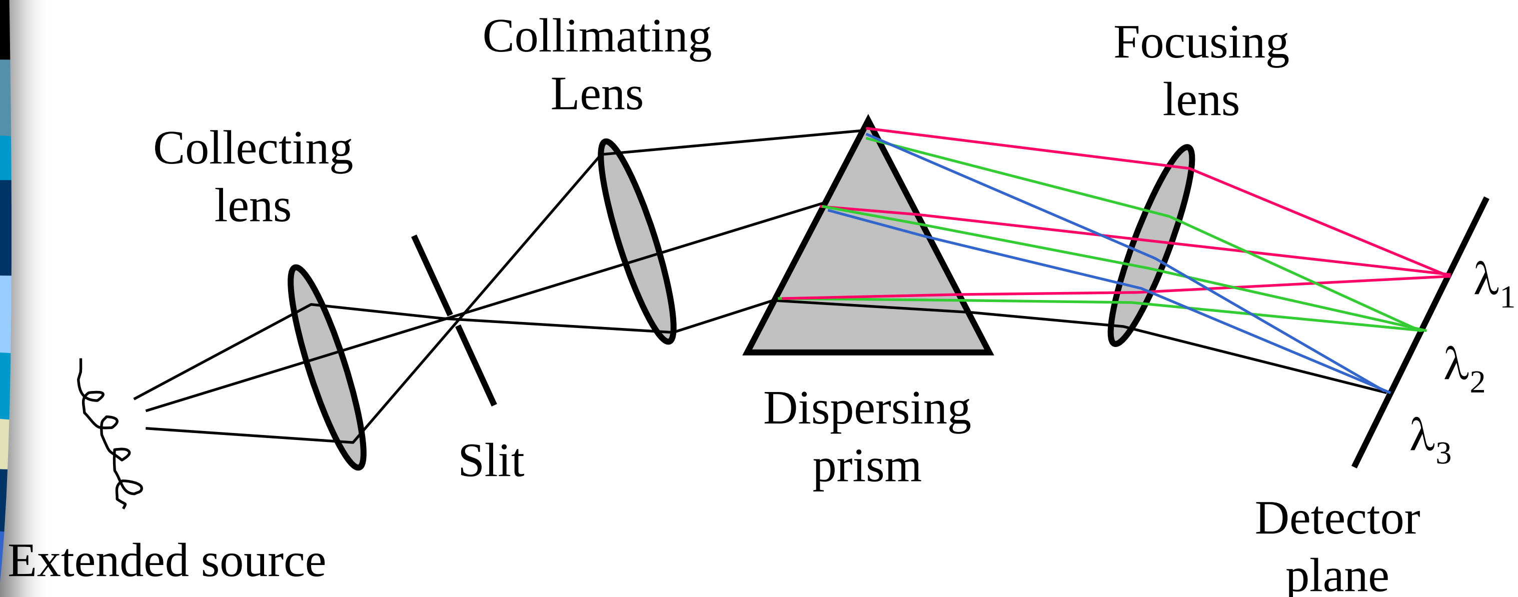
- A spectroscopic instrument that may scan wavelengths individually or the entire spectra simultaneously.

- It may employ a **prism** or **grating** for means of dispersing incident light.



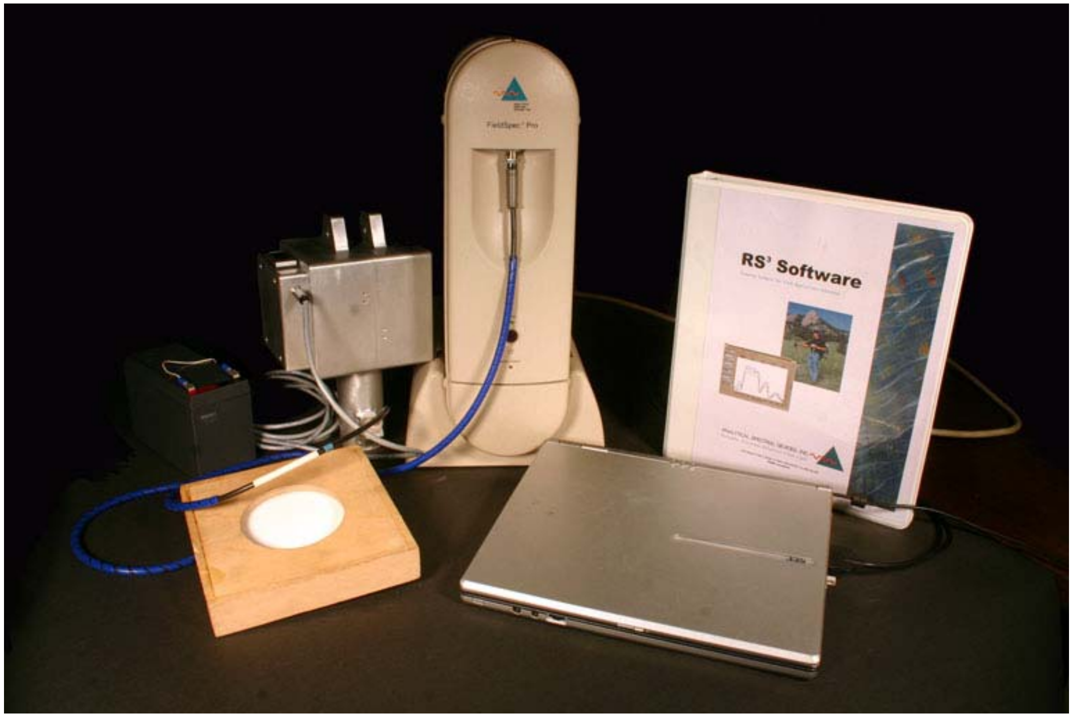
Prism Spectrometer

- Using prism and dispersion
- Light must be collimated onto prism



Grating Spectrometer

- High dispersion using compact element
- The element used in such a systems is:
  - A diffraction grating
- Comes in many configurations



Spectrometer Applications

- Is used in spectroscopy (the study of spectra)
  - Producing spectral lines
  - Measuring their wavelengths and intensities
- Astronomy
  - Most large telescopes have spectrographs
  - Measure chemical compositions of objects
  - Measure velocities from the Doppler shift of spectral lines
- Remote Sensing
  - Imaging spectrometers (hyperspectral data)
  - Field spectroscopy of natural and man-made objects

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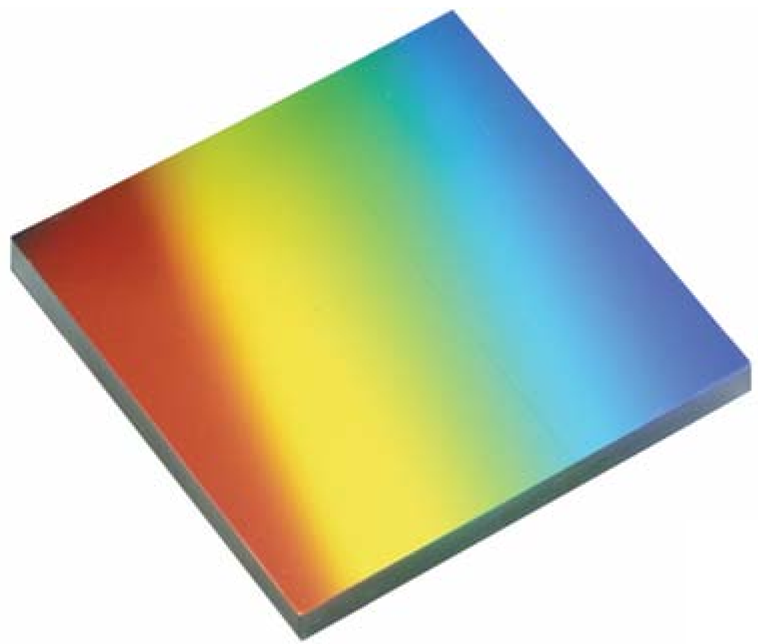
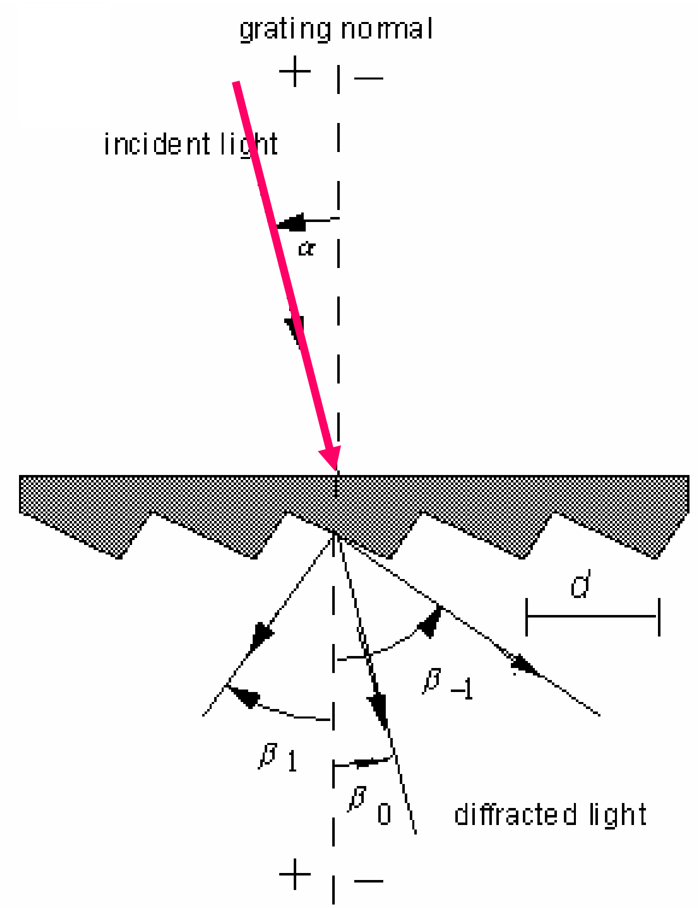
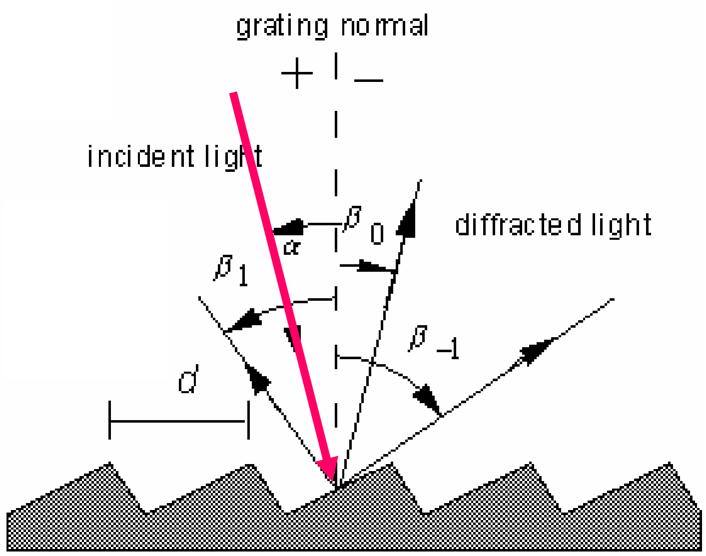
Diffraction Grating Types

- Reflection grating
  - Grating superimposed on a reflective surface
- Transmission grating
  - Grating superimposed on a transparent surface



Diffraction by Plane Grating

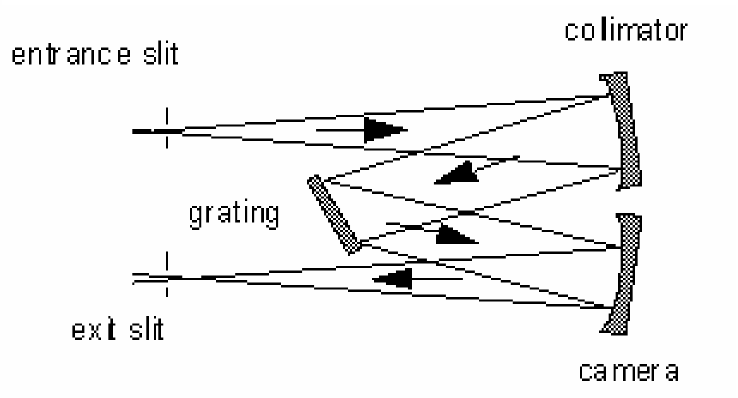




-Incident light, α, diffracted light, β_{-1}, β_{0}, β_{1}

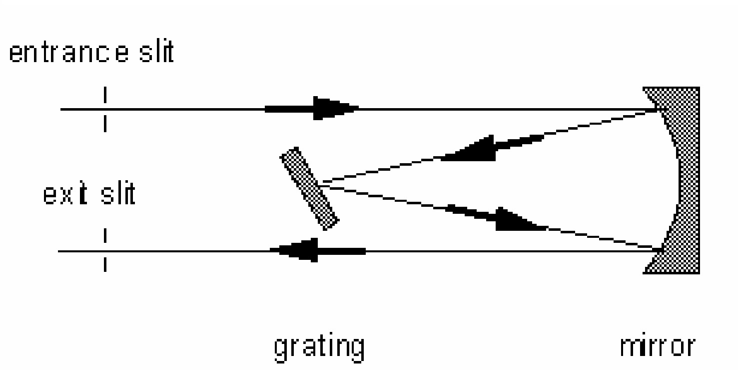
-Think of each grating groove as being a very small, slit-shaped source of diffracted light
-Angles are measured FROM the grating normal TO the beam

Plane Gratings



Czerny-Turner

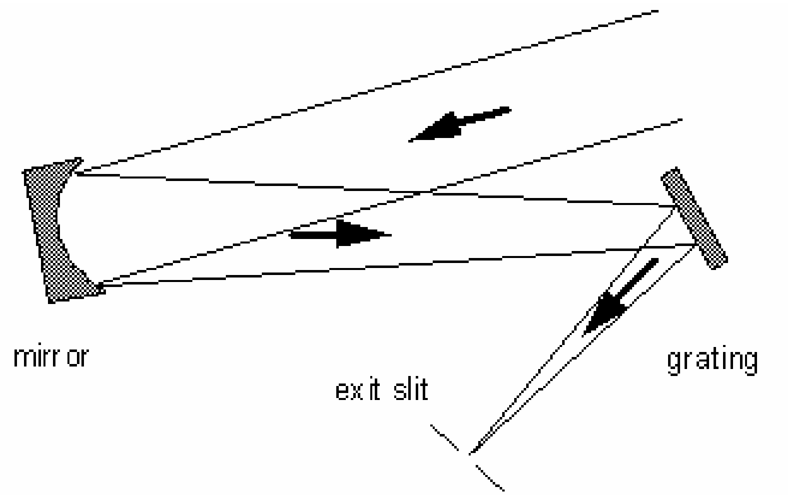
- Illuminated by collimated light
- No aberrations introduced into the diffracted wave fronts



Ebert-Fastie

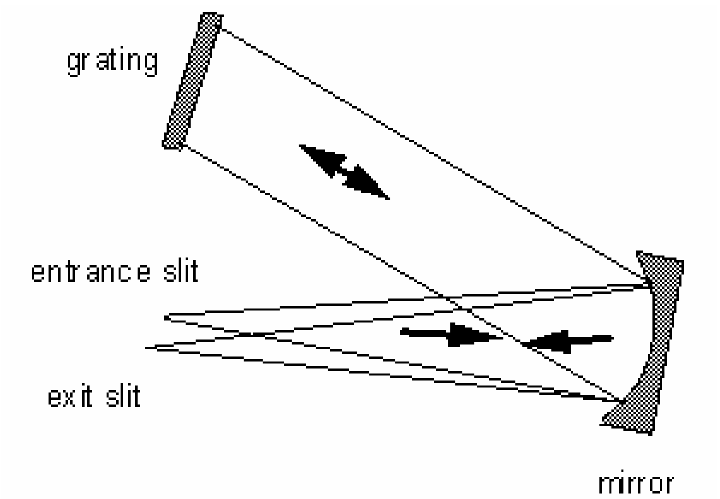
- Large mirror serves as both collimator and camera
- Use is limited since stray light and aberrations are difficult to control

Plane Gratings



Monk-Gillieson

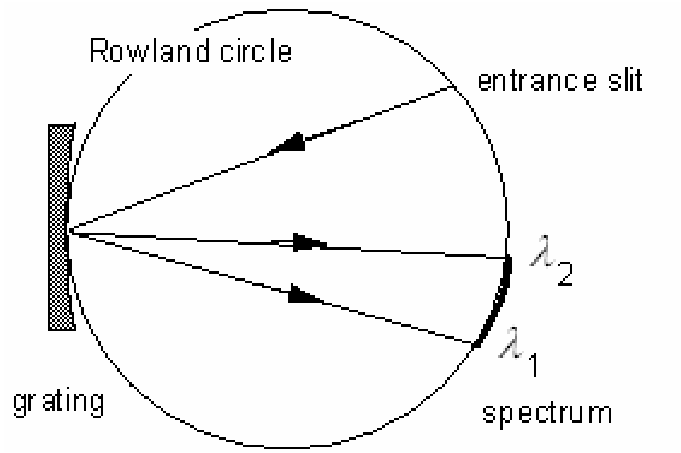
- Illuminated by converging light
- Aberrations introduced into the diffracted wavefronts
- Good for low-resolution applications
- Simplest and least expensive design



Littrow

- Autocollimating configuration

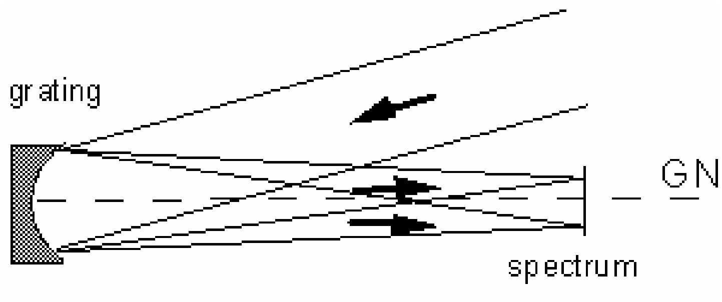
Concave Gratings



Rowland Circle Spectrograph

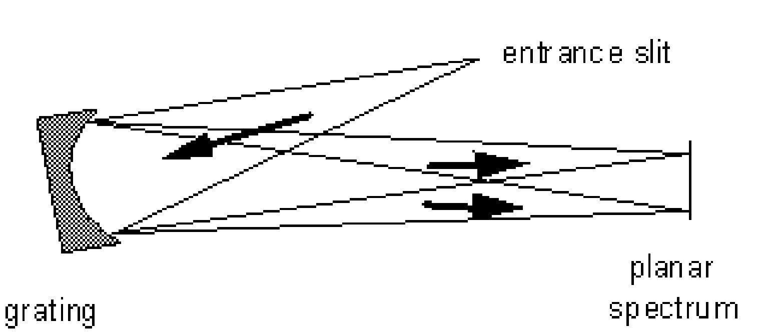
- Illuminated by point source on circle
- Spectra on circle is free from defocus and primary coma at all wavelengths

Concave Gratings



Wadsworth spectrograph

- Illuminated with collimated light



Flat-field spectrgraph

- Forms a spectrum on a flat surface
- Ideal for use in linear detector array instruments

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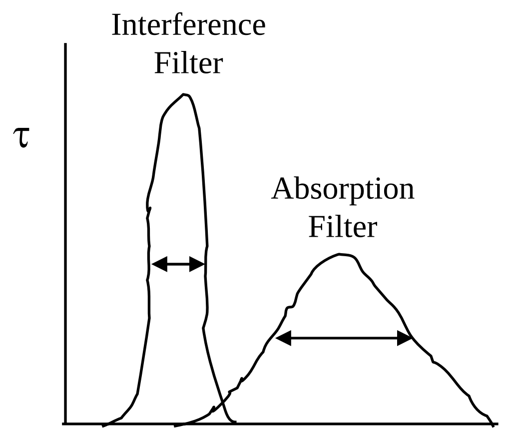
- Interferometers

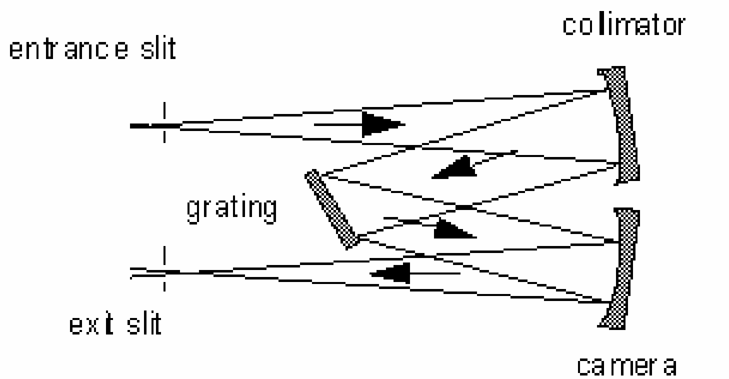
Wavelength Selectors

- Ideally want to isolate 1 wavelength
- End up with a narrow band
- Two types encountered
  - Filters
    - Absorption
      - Limited to VIS
    - Interference (Faby-Perot)
      - Used in UV, VIS, IR
  - Monochromators



FWHM: 10nm and 50nm

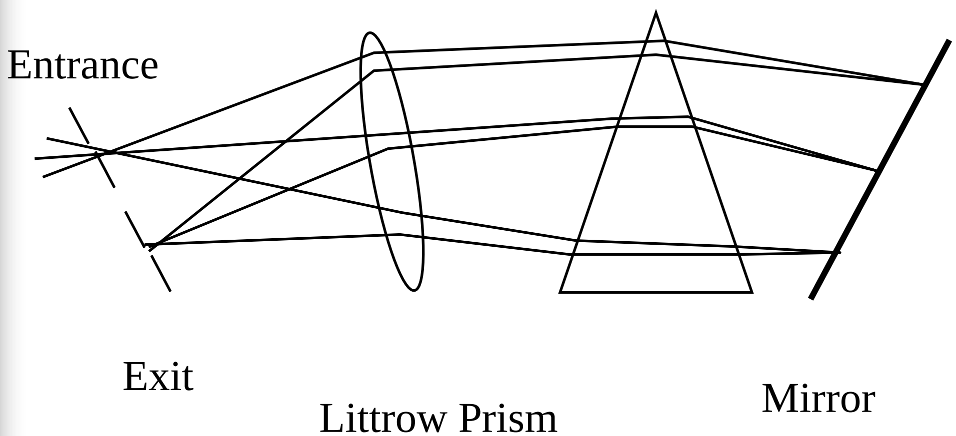
Monochromators



Czerny-Turner

• Come in both **prism** and **grating** types

•At output we get rectangular images of the entrance slit



Littrow Prism