SIMG-320 – Linear Mathematics for Imaging

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Office Hours TBD, and by appointment

Webpages
Course: http://www.cis.rit.edu/class/simg320/index.html
My current schedule: http://www.cis.rit.edu/people/faculty/easton/Easton_Schedule_20062.pdf

Meeting Rooms/Times: TTh, 2:00PM – 3:50PM , Room 76-1230
I hope to schedule additional OPTIONAL times for group problem sessions.

Prerequisites:
Calculus I - IV

Details:
Homework will be assigned, and is to be handed in on time (adjustments will be considered in advance). Problems will be graded and solutions handed out.

Homework **–** 30% (Assignments usually given Tuesdays, due Thursday 1 week later AT BEGINNING OF class.)
Midterm Exam (anticipated date: Th 1/18/2006) **–** 30%
Final Exam (cumulative) **–** 40%

My philosophy on exams: they are intended to test *understanding* of material, which is the ability to assimilate concepts and synthesize useful results for applications. This is *not* the same as the ability to parrot discussions of concepts or replicate the solutions to homework problems. I particularly like problems that appear to be difficult but actually are easy if you see the connection (my exams seem to have a reputation among students).

Subject:
This course introduces mathematical tools for describing imaging systems. In other words, the course develops mathematical models of imaging systems and applies them to problems relevant in imaging.

Text materials:
Many books are available that touch on aspects of mathematical models of imaging systems and parameters, though the appropriate single book which encompasses all (or even most) aspects at the appropriate level has yet to be published. For now, the text materials come from the manuscript *Linear Systems with Applications to Imaging*, which have been copied and will be sold at cost. Of course, many other text resources are available (perhaps more than you want to know about!), including:
Linear Systems, Fourier Transforms, and Optics, Jack D.Gaskill, Wiley, 1978, QC355.2.G37, ISBN 0-471-29288-5
Unfortunately, Gaskill's book does not give sufficient consideration to the derivation and application of discrete (sampled) systems. This subject is covered to some extent in my book and other resources are listed on subsequent pages.

Mathematical Foundations of Linear Systems:

1. For review, Schaum's Outlines on Calculus, Linear Algebra, Vector Analysis, Matrices, and Complex Variables and Schaum's *Mathematical Handbook*
2. *Advanced Mathematical Methods for Engineering and Science Students*, G. Stephenson, P.M.Radmore, Cambridge, 1990 (§2 on special functions, §7 on Fourier transforms).
3. *Linear Algebra and its Applications* (3rd Edition), Gilbert Strang, Harcourt,Brace,Jovanovitch, 1988, (Chapters on orthogonal projections, eigenvectors, change of bases).
4. Any of several books on mathematical physics, *e.g.*, Kreysig, Arfken, Byron and Fuller, ...

Fourier Transforms in Mathematics:

1. *The Fourier Integral and Certain of its Applications*, N.Wiener, Dover Publications, 1958 (first published in 1933 -- *tediously mathematical*), QA404.W47.
2. *An Introduction to the Theory of Fourier's Series and Integrals*, H.S. Carslaw, Dover Publications, 1950 (first published in 1930 -- *also mathematical, but easier to read than Wiener)* QA404.C32
3. *A Handbook of Fourier Theorems*, D.C. Champeney, Cambridge, 1987, (*best of the three*) QA403.5.C47.

Fourier Transforms in Physics/Engineering:

1. *Fourier Series and Boundary-Value Problems*, R.V.Churchill, McGraw-Hill, 4th Edition, 1987, (*classic text with lots of physical applications*), QA404.C6.
2. *A First Course in Fourier Analysis*, D.M. Kammler, Prentice-Hall, 2000, (useful discussions of mathematical and computational aspects), QA403.5.K36.
3. *Fourier Transforms and their Physical Applications*, D.C.Champeney, Academic Press, 1973, *(excellent)*, QA403.5.C46.
4. *Fourier methods for mathematicians, scientists, and engineers*, M.Cartwright, Ellis Horwood, 1990, *(paperback, introductory, lots of physical applications)*, QA403.5.C37.
5. *The Fourier Transform and Its Applications* (Second Edition, Revised), R.N.Bracewell, McGraw-Hill, 1986, *(the standard reference on 1-D Fourier, good discussion of discrete transforms and applications)*, QA403.5.B7
6. *Fourier Transforms, An Introduction for Engineers*, R.M.Gray and J.W.Goodman, Kluwer Academic Publishers, 1995, *(aimed at discrete transform, not as useful as I expected)*, TK5102.9.G73.
7. *A student's guide to Fourier transforms*, J.F.James, Cambridge, 1995, QC20.7.F67J36, (*thin, cheap, useful)*
8. *The Fourier Integral and its Applications*, A. Papoulis, McGraw-Hill, 1962, *(old - preFFT, though good mix of mathematical theory and practical applications)*, QA404.P32.
9. *Fourier Transforms*, I.N.Sneddon, Dover Publications, 1995 (first published in 1951), *(similar comments to Papoulis)*, QA404.S53.
10. *Fourier Analysis*, T.W.Körner, Cambridge, 1988, *(potpourri of Fourier from nonconventional point of view -- historically driven)*, QA403.5.K67.
11. *Exercises for Fourier Analysis*, T.W.Körner, Cambridge, 1993, *(see comment above)* ,QA403.5.K66.
12. *Integral Transforms in Science and Engineering*, K.B.Wolf, Plenum, 1979, *(mathematical reference)*, QA432.W64.
13. *Probability, Statistical Optics, and Data Testing*, 2nd Ed. B.R.Frieden, Springer-Verlag, 1991 (particularly §4 on Fourier methods, *(excellent discussion of applications of statistical principles to many types of imaging problems, not just optics)*, QA273.F89.
14. *Statistical Optics*, J.W. Goodman, Wiley, 1985, *(applications of Fourier theory to statistics, particularly in optics)*, QC355.2.G66.

15. *Who is Fourier? A Mathematical Adventure*, Transnational College of LEX, Language Research Foundation, 1995. *($25 paperback translated from Japanese, very introductory, lots of pictorial examples. usefulness limited by lack of index).*
16. *The Hartley Transform*, R.N.Bracewell, Oxford, 1986, (*special case of Fourier transform, a favorite of the author)* QA403.5.B73.

Discrete Fourier Transforms:

1. *The FFT, Fundamentals and Concepts*, R.W.Ramirez, Prentice-Hall, 1985, *(graphical introduction to discrete Fourier transform)*.QA403.5.R36.
2. *The Fast Fourier Transform and its Applications*, E.O.Brigham, Prentice-Hall, 1988, *(excellent)*, QA403.B75.
3. Fast Fourier Transforms, J.S.Walker, 2nd Edition, CRC Press, 1996, *(new edition, includes software)*, QA403.W33.
4. *Multidimensional Digital Signal Processing*, D.E.Dudgeon and R.M.Mersereau, Prentice-Hall, 1984 (§1-2), *(written for EEs, but good discussion of 2-D discrete transform,* TK5102.5.D83.
5. *Digital Image Processing*, K.R.Castleman, Prentice-Hall, 1996 (§1-2,§9-16), *(excellent for lots of imaging problems, demonstrates relationship of linear systems to optical systems)*,TA1632.C37.

Linear Systems and Optical Imaging:

1. *Introduction to Fourier Optics*, J.W.Goodman, (2nd Edition), McGraw-Hill, 1996, *(updated classic)*, QC355.G65.
2. *Fourier Optics, An Introduction* (2nd Edition), E.G.Steward, Wiley, 1987, *(useful introduction, lower level than Goodman)*, QC454.F7S83.
3. *Introduction to the Optical Transfer Function*, C.S.Williams and O.A.Becklund, Wiley, 1989, *(specialized topic of linear systems in optics)*, QC367.W55.
4. *Systems and Transforms with Applications in Optics*, A.Papoulis, McGraw-Hill, 1968, *(another classic, showing its age a bit)*, QC383.P23.
5. *Applications of Optical Fourier Transforms*, H.Stark, ed., Academic Press, 1982, *(as implied, discussions of specific applications)*, TA1632.A68.
6. *Quantitative Coherent Imaging: Theory, Methods, and Some Applications*, J.M.Blackledge, Academic Press, 1989, *(nice desciption of subject, unusual notation/spellings)*, QC476.C6.B553
7. *The New Physical Optics Notebook*, Reynolds, DeVelis, Parrent, and Thompson, SPIE Press, 1989, *(applications of linear systems to optics/holography)*, QC395.2.N48.
8. *Fourier Series and Optical Transform Techniques in Contemporary Optics*, Raymond Wilson, John Wiley & Sons, Inc, 1995. QC454.F7 W55 (ISBN 0-471-30357-7)

Image Recovery:

1. *Image Restoration and Reconstruction*, R.H.T.Bates and M.J.McDonnell, Oxford University Press, 1986, *(application of linear systems to imaging)*, TA1632.B36.
2. *Image Recovery, Theory and Application*, (H.Stark, ed.), Academic Press, 1987, *(similar to Bates but more applications, multiple authors, fragmented)*, TA1632.I4824.

Useful References from Magazines and Journals:

1. "The Fourier Transform", R.N. Bracewell, in *Scientific American* June 1989, pp.86-95.
2. "Numerical Transforms", R.N. Bracewell, in *Science*, v.248 11 May 1990, pp.697-704.
3. "Fourier Analysis Using a Spreadsheet", R.A. Dory and J.H. Harris, in *Computers in Physics* Nov.-Dec. 1988, pp. 83-86.
4. "A Plain Man's *(sic)* Guide to the FFT", P. Kraniauskas, in *IEEE Signal Processing Magazine*, v. 11, April 1994, pp. 24-35.
5. "Tom, Dick, and Mary Discover the DFT", J.R. Deller, Jr., in *IEEE Signal Processing Magazine*, v. 11 April 1994, pp. 36-50.

6. "SIGNALS, Interactive Software for One-Dimensional Signal Processing, R.L. Easton, Jr., in *Computer Applications in Engineering Education*, v.1 December 1993, pp.489-501.

7. "Fast Fourier Transforms For Fun and Profit", W.M. Gentleman and G. Sande, in *Proceedings - Fall Joint Computer Conference*, 1966, pp.563-578.

Other books containing useful discussions of imaging subjects:

1. *Principles of Digital Image Synthesis*, Andrew Glassner, Morgan-Kauffman, 1995 (two volumes), *(very nice discussion of broad range of imaging topics, relevant material in §4-5,§8-10*), T385.G585.
2. *Image Reconstruction in Radiology*, J. Anthony Parker, CRC Press, 1990, (*of much more general application than the title indicates; written for medical students and radiologists, does not require a "high" level of mathematical knowledge, useful intuitive discussions of imaging principles*) RC78.7.D53 P36.
3. *Radiological Imaging*, H.H. Barrett and W.Swindell, Academic Press, 1981, *(terrific book, also much more general than indicated by its title)*, RC78.B337, (§2, §4 on Linear Systems, §3 on Random Processes, §7 on Computed Tomography)

Computing Resources:

Many computational software packages are available that would be helpful when learning the material in this class. CIS has selected ***IDL™*** from RSI as its "standard" package. It is installed on the *UNIX* workstations in the Center, and also is available for purchase from CIS; the price for a full working copy is $200 (or thereabouts), *vs.* the list price of »$1500. Other packages exist, including ***Mathematica™*** (available on RIT VAX), ***MathCad™***, ***Matlab™***, and ***Scientific Workplace™***. All these packages allow computations involving most aspects of matrix algebra and complex analysis to be evaluated quickly and (more or less) painlessly. They also have graphing routines which may assist in visualizing concepts. In my opinion, most of the packages have a fairly steep learning curve -- you can't do much that is useful very quickly. The programs also have their respective advantages and disadvantages, *e.g.*, my opinion is that the interfaces to ***Mathematica™*** and ***MathCAD™*** are not very intuitive, which means that new users have to travel the learning curve. Conversely, experienced users are rewarded by quicker answers.

For some specific applications in 1-D linear systems, my program for PCs ("***SIGNALS***") may be useful. It was written with the intent of being easy to use, though you must decide for yourself if it succeeds. It is available *gratis* if you supply the diskette, and is installed on the PCs in the CIS computing complex (click on the ***SIGNALS*** icon or type "signals" at the command prompt) and may be downloaded from the *Blackboard* site or from the CIS website at:

http://www.cis.rit.edu/resources/software/index.html

The user manual also is available online at:

http://www.cis.rit.edu/resources/software/sig_manual/index.html

SIMG-320 – Linear Mathematics for Imaging

This course presents mathematical descriptions for functions and systems and demonstrates their application to solving imaging problems. Alternative descriptions of images and imaging systems will be derived that are conceptually based on the *projection* of one vector onto another, which leads naturally to the concepts of *orthogonal* vectors and orthogonal *functions*.

Course Outline:

- I. Introduction and motivation
  - A. The Imaging "Chain"
  - B. Mathematical expression for an imaging system: $\mathcal{O}\{f[x,y,...]\} = g[x,y,...]$
  - C. The three imaging "tasks" (direct problem, inverse problem, system analysis/synthesis)
  - D. Examples of imaging systems and mathematical models
    - 1. "imaging" of optical rays
      - a. imaging without a lens
      - b. "imaging" of optical rays by selection with pinhole
      - c. multiple pinholes
    - 2. "redirection" of rays by mirror or lens
    - 3. Examples of "imaging tasks" in medicine
      - a. Gamma-ray imaging
      - b. Radiography
      - c. Computed Tomographic Radiography (CT)
    - 4. Image "quality"
  - E. Necessity to constrain possible action of system for mathematically tractible description
- II. Functions
  - A. Continuous and DiscreteDomains
  - B. Continuous and Discrete Ranges
  - C. Discrete Domain and Range, "Digital" Functions
  - D. Periodic, Aperiodic, and Harmonic Functions
  - E. Symmetry Properties of Functions
- III. Vector and Matrix Concepts
  - A. Scalars and vectors with real-valued components
    - 1. vector addition
    - 2. scalar multiplication
    - 3. triangle inequality
    - 4. scalar (dot) product
      - a. length (norm)
      - b. projection of one vector onto another
      - c. Cauchy-Schwarz inequality
    - 5. Matrices as multiple scalar products
      - a. matrix-vector product
      - b. matrix-matrix product
      - c. square matrices
      - d. diagonal matrices
      - e. identity matrix
    - 6. vector spaces
    - 7. basis vectors
      - a. Constructing different sets of basis vectors
      - b. Gram-Schmidt orthogonalization

c. rotation of vectors

IV. Complex numbers
- A. as real-valued vectors
- B. representations
  1. real/imaginary parts
  2. magnitude/“phase”
- C. Graphical representation on phasor/Argand diagram
- D. complex arithmetic
- E. Euler relation, deMoivre's theorem
- F. Complex functions of a real variable, e.g., $f[x]$
- G. Complex functions of a complex variable, e.g., $w[z]$
  1. examples, e.g., z^{+1}, z^{*}, z^{-1}
  2. Introduction to path integrals

V. Vectors with complex-valued components
- A. inner product
  1. norm (length)
  2. projections
- B. Operators on vectors, matrices
  1. matrix-vector multiplication as multiple scalar products or as simultaneous linear equationsrepresentions of arbitrary vectors, projection onto different basis sets
  2. imaging problems in matrix-vector form
     - a. matrix inverses
     - b. pseudoinverses
     - c. shift invariance, circulant matrices

VI. Eigenvectors and Eigenvalues
- A. diagonal forms of matrix operator
- B. diagonalization operators
- C. diagonalization of circulant matrix
- D. discrete Fourier transform (DFT)

VII. Matrix-Vector Formulations of the Imaging “Tasks”
- A. Inverse Task
  1. matrix inverse
  2. Solution of inverse problem by diagonalization
- B. Matrix-vector formulation of system analysis
- C. Alternative representation of shift invariant system, rotation of basis vectors

VIII. Functions of continuous variables $f[x]$
- A. Classification
  1. domain and range (real/complex, continuous/discrete)
  2. form (linear/nonlinear, periodic, harmonic)
  3. symmetry (even/odd)
- B. Representations obtained by projecting onto different sets of basis functions
  1. inner product, relation to scalar product
  2. orthogonal/orthonormal sets of functions
  3. power series representations, Taylor series
- C. Representations of functions
  1. real/imaginary parts
  2. magnitude (modulus)/phase
  3. Argand-phasor diagram (Lissajou figures)

IX. Continuous-Domain Analogues of VectorOperations

A. Inner Product of Continuous Functions
B. Projections of Continuous Functions
C. Special Functions
   a. 1-D real-valued functions (constant, RECT, TRI, SGN, COS, CHIRP, GAUS)
   b. 1-D Dirac delta function (impulse) and related functions
   c. 1-D complex-valued sinusoid

X. Mathematical representation of systems, operators
   A. “Linearity”
   B. “Shift (space-, time-) invariance”
   C. Linear and shift-invariant (LSI) systems, action of system is convolution (filtering)
   D. Representations of systems
      1. linear and discrete (matrix-vector multiplication)
      2. linear and continuous (superposition integral)
      3. LSI and discrete (circulant matrix, diagonalizing transformation)
      4. LSI and continuous (convolution integral)
      5. impulse response/point-spread function as descriptor of system
   E. Crosscorrelation and autocorrelation

XI. Fourier transforms of 1-D continuous functions
   A. Direct integration
   B. Fourier transforms of special functions
   C. Theorems of the Fourier transform