1051-320-20062 MIDTERM Exam Closed Book

Do all problems. **Show your work**. If you solve problems "by inspection", then write down your thought sequence that led to your conclusion. Point totals as indicated:

1. (30%) The first row of a 3×3 "circulant matrix" $\underline{\mathbf{A}}$ is:

$$\begin{bmatrix} +1 & +1 & +1 \end{bmatrix}$$

   (a) Write down the complete matrix $\underline{\mathbf{A}}$.

   (b) Write down an expression for the vectors that are in the "row subspace." of $\underline{\mathbf{A}}$.

   (c) Write down an expression for the vectors that are in the "column subspace." of $\underline{\mathbf{A}}$.

   (d) Describe the vectors that are in the "null subspace" of $\underline{\mathbf{A}}$; your description can be an equation similar to that in part (b) or a sketch or a description in words.

   (e) Does $\underline{\mathbf{A}}^{-1}$ exist? Explain your answer.

2. (20%) For each of the following vectors, find ONE orthogonal vector *with unit length.*

   (a) $\underline{\mathbf{x}}_1 = \begin{bmatrix} 1 \\ -0.5 \\ -0.5 \end{bmatrix}$

   (b) $\underline{\mathbf{x}}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

3. (20%) Find ALL distinct complex numbers z that are solutions of the equation below; express them as real and imaginary parts AND as magnitude and phase (you may leave the values in the form of trigonometric functions, e.g., $\cos\left[\frac{\pi}{32}\right]$, or you may evaluate them).

$$z^4 = -i$$

4. (10%) Find the "magnitudes" (i.e., the "lengths") of the following vectors:

   (a) $\underline{\mathbf{x}}_1 = \begin{bmatrix} 1 \\ -0.5 \\ -0.5 \end{bmatrix}$

   (b) $\underline{\mathbf{x}}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

MORE→→→

5. (10%) Find the phase angles of the following complex numbers:

(a) $z_a = 0 - 10i$

(b) $z_b = -4 + 4i$

6. (10%) In each case, calculate the "projection" of the vector $\underline{\mathbf{x}}$ onto the vector $\underline{\mathbf{a}}$,

(a) $\underline{\mathbf{x}} = \begin{bmatrix} +1 \\ +1 \end{bmatrix}, \quad \underline{\mathbf{a}} = \begin{bmatrix} +2 \\ +2 \end{bmatrix}$

(b) $\underline{\mathbf{x}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \underline{\mathbf{a}} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$