

1. Write a program in a computer language of your choice (e.g., IDL) to evaluate the DFT of an 8-element vector, e.g.,

$$\underline{\mathbf{x}} = \begin{bmatrix} (\underline{\mathbf{x}})_0 \\ (\underline{\mathbf{x}})_1 \\ (\underline{\mathbf{x}})_2 \\ (\underline{\mathbf{x}})_3 \\ (\underline{\mathbf{x}})_4 \\ (\underline{\mathbf{x}})_5 \\ (\underline{\mathbf{x}})_6 \\ (\underline{\mathbf{x}})_7 \end{bmatrix} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix}$$

The expression for the n-dimensional DFT is:

$$\underline{\mathbf{x}}' = \underline{\mathbf{D}}^{-1} \underline{\mathbf{x}}$$

$$(\underline{\mathbf{x}}')_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \exp \left[\left(+2\pi i \frac{nk}{8} \right) \right]^* (\underline{\mathbf{x}})_n = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \exp \left[-2\pi i \frac{nk}{8} \right] \cdot x_n$$

In words, the k^{th} component of the output vector $\underline{\mathbf{x}}'$ is the weighted sum of the 8-elements of the input vector $\underline{\mathbf{x}}$ where the weight factor applied to the n^{th} component of $\underline{\mathbf{x}}$ is $\exp \left[-2\pi i \frac{nk}{8} \right]$.

2. Use the program to evaluate the DFT of the following 8-element vectors:

$$\underline{\mathbf{x}}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \underline{\mathbf{x}}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \underline{\mathbf{x}}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \underline{\mathbf{x}}_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \underline{\mathbf{x}}_5 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \\ -1 \\ -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \quad \underline{\mathbf{x}}_6 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

3. For each case, plot graphs of both the input and output vectors $\underline{\mathbf{x}}_m$ and $\underline{\mathbf{x}}'_m$. In other words, the index $n = 0, 7$ is plotted on the horizontal axis and the real part and imaginary part of the amplitudes are plotted along the vertical axis. I suggest that you use some computer graphing program, e.g., IDL.
4. OPTIONAL EXTRA CREDIT: Extend your program to evaluate the DFT of a 64-element vector and use it to evaluate the DFTs of the following vectors:

- (a) $(\underline{\mathbf{x}})_n = x_n = 1$
- (b) $x_n = \cos \left[2\pi \frac{n}{64} \right]$
- (c) $x_n = \cos \left[2\pi \frac{n}{8} \right]$
- (d) $x_n = \sin \left[2\pi \frac{n}{8} \right]$