In the book, read the sections in Chapter 6 about the functions we have discussed and read Chapter 8.

Do the following problems:

1. Sketch the following functions and evaluate their continuous Fourier transforms

(a) \( f_1 [x] \equiv \begin{cases} 0 & \text{if } |x| > 2 \\ \frac{1}{2} & \text{if } |x| = 2 \\ 1 & \text{if } |x| < 2 \end{cases} \)

(b) \( f_2 [x] \equiv \begin{cases} 0 & \text{if } x > 4 \\ \frac{1}{2} & \text{if } x = 4 \\ 1 & \text{if } 0 < x < 4 \\ \frac{1}{2} & \text{if } x = 0 \\ 0 & \text{if } x < 0 \end{cases} \)

(c) \( f_3 [x] = \exp[-x] \cdot STEP [x] \)

(d) \( f_4 [x] = \exp[-x] \cdot STEP \left[ \frac{x}{2} \right] \)

2. The operation of “convolution” of two functions \( f [x] \) and \( h [x] \) is defined:

\[
f [x] * h [x] = \int_{-\infty}^{+\infty} f [\alpha] \cdot h [x - \alpha] \, d\alpha
\]

Evaluate the convolution of \( f [x] = \cos \left[ 2\pi \frac{x}{2} + \frac{\pi}{4} \right] \) with the following functions for \( h [x] \) and sketch (or plot) the results:

(a) \( h_1 [x] = RECT [x] \)

(b) \( h_2 [x] = \frac{1}{2} RECT \left[ \frac{x}{2} \right] \)

3. Evaluate the convolutions of the following functions by “direct integration” and sketch (or plot) the results:

(a) \( g_1 [x] = RECT [x] * RECT [x] \)

(b) \( g_2 [x] = RECT \left[ \frac{x}{2} \right] * RECT \left[ \frac{x}{2} \right] \)

(c) \( g_3 [x] = RECT [x] * RECT [x] * RECT [x] \)

4. Evaluate and sketch (or plot) the following convolutions:

(a) \( (\exp[-x] \cdot STEP [x]) * RECT [x] \)

(b) \( (\exp[-x] \cdot STEP [x]) * (\exp[-x] \cdot STEP [x]) \)