In these problems, you may (and are encouraged to!) use a computer mathematical toolbox (such as Mathematica or Matlab). Such toolboxes are available on the PCs in room 76-1225

1. For a general 4×4 circulant matrix $\underline{\mathbf{A}}$:

   (a) Derive (or just write down) the diagonalizing matrix $\underline{\mathbf{D}}$

   (b) Derive (or just write down) the inverse matrix $\underline{\mathbf{D}}^{-1}$ and demonstrate that $\underline{\mathbf{D}}^{-1}\underline{\mathbf{D}} = \underline{\mathbf{I}}$.

   (c) Make graphs of the magnitude **AND** of the phase of the four eigenvectors of the general 4×4 circulant matrix $\underline{\mathbf{A}}$ on a plot where the horizontal axis has four coordinates labeled by the number k of oscillation cycles of the eigenvectors per N sample intervals. In other words, the four axis coordinates are labeled by $k = 0,\ k = 1,\ k = 2$, and $k = 3$.

Use the matrices $\underline{\mathbf{D}}$ and $\underline{\mathbf{D}}^{-1}$ in the following:

2. The first row of a 4×4 circulant matrix $\underline{\mathbf{A}}$ is:

$+\frac{1}{2}$	$+\frac{1}{2}$	0	0

   (a) Construct the matrix $\underline{\mathbf{A}}$.

   (b) In your own words, describe what $\underline{\mathbf{A}}$ does to an input vector $\underline{\mathbf{x}}$.

   (c) Find the eigenvalues of $\underline{\mathbf{A}}$ and derive (or just write down) the diagonal form $\underline{\mathbf{\Lambda}}$.

   (d) Determine if $\underline{\mathbf{A}}^{-1}$ exists and explain how you know.

   (e) Determine which eigenvectors of $\underline{\mathbf{A}}$ (if any) are in the null subspace of $\underline{\mathbf{A}}$.

   (f) Find $\underline{\mathbf{A}}^{-1}$ if it exists or $\underline{\mathbf{A}}^{\dagger}$ if $\underline{\mathbf{A}}^{-1}$ does not exist.

3. Repeat #2 if the first row in the 4×4 circulant matrix $\underline{\mathbf{A}}$ is:

$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	0

4. Repeat #2 if the first row in the 4×4 circulant matrix $\underline{\mathbf{A}}$ is:

$+\frac{1}{4}$	$+\frac{1}{4}$	$+\frac{1}{4}$	$+\frac{1}{4}$

5. Repeat #2 if the first row in the 4×4 circulant matrix $\underline{\mathbf{A}}$ is:

-1	$+1$	0	0