1051-320-20062 Homework #5 Due 1/30/2007 (T)

1. In each case, calculate the "projection" of the vector $\underline{\mathbf{x}}$ onto the vector $\underline{\mathbf{a}}$:

   (a) $\underline{\mathbf{x}} = \begin{pmatrix} i \\ i \\ 0 \end{pmatrix}$, $\underline{\mathbf{a}} = \begin{pmatrix} 0 \\ i \\ i \end{pmatrix}$

   (b) $\underline{\mathbf{x}} = \begin{pmatrix} 1+i \\ 1 \\ 1-i \end{pmatrix}$, $\underline{\mathbf{a}} = \begin{pmatrix} 1 \\ 1+i \\ 1-i \end{pmatrix}$

   (c) $\underline{\mathbf{x}} = \begin{pmatrix} \frac{1}{2} \\ \frac{i}{2} \\ -\frac{1}{2} \\ -\frac{i}{2} \end{pmatrix}$, $\underline{\mathbf{a}} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

2. Find a *unit vector* that is orthogonal to each of the following 2-D vectors:

   (a) $\underline{\mathbf{v}}_1 = \begin{pmatrix} -1+i \\ +1-i \end{pmatrix}$

   (b) $\underline{\mathbf{v}}_2 = \begin{pmatrix} 1 \\ i \end{pmatrix}$

   (c) $\underline{\mathbf{v}}_3 = \begin{pmatrix} e^{+i\frac{\pi}{3}} \\ e^{-i\frac{\pi}{4}} \end{pmatrix}$

3. Consider these "imaging system" matrices $\underline{\mathbf{A}}$ that act on 2-D vectors:

$$\underline{\mathbf{A}}_1 = \begin{bmatrix} 2i & 1 \\ 1 & 2i \end{bmatrix}, \quad \underline{\mathbf{A}}_2 = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}, \quad \underline{\mathbf{A}}_3 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

   (a) In each case, find the "diagonalized" matrix $\underline{\mathbf{\Lambda}}_n$ corresponding to $\underline{\mathbf{A}}_n$.

   (b) From the forms of $\underline{\mathbf{\Lambda}}_n$, determine which of the matrices $\underline{\mathbf{A}}_n$ are invertible.

   (c) Find the inverses $\underline{\mathbf{A}}_n^{-1}$ that exist and the pseudoinverses $\underline{\mathbf{A}}_n^{\dagger}$ in those cases where $\underline{\mathbf{A}}_n^{-1}$ does not exist.

   (d) Determine normalized basis vectors for the row subspace, null subspace, column subspace, and left-null subspace.

4. The first rows of several circulant matrices are listed. In each case, construct the matrix and find the eigenvectors and corresponding eigenvalues of each.

   (a) $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$

   (b) $\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$

   (c) $\begin{bmatrix} +\frac{1}{2} & +\frac{1}{2} & 0 & 0 \end{bmatrix}$

   (d) $\begin{bmatrix} -1 & +1 & 0 & 0 \end{bmatrix}$