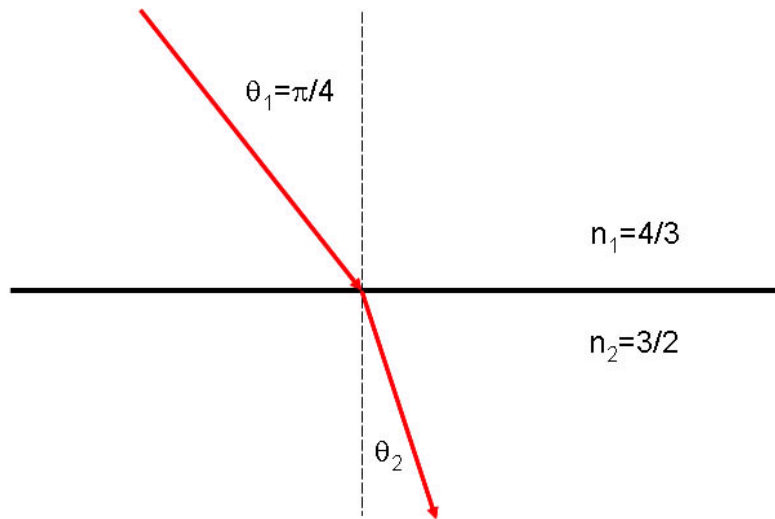


SIMG-303-20033 Solution Set #4

1. Given an interface between water ($n_{water} = \frac{4}{3}$) and glass ($n_{glass} = \frac{3}{2}$), compute the transmission angle for a beam incident in the water at 45° . If the transmitted beam is reversed so that it impinges on the interface at the same angle that it was transmitted, show that the transmission angle is 45° .

$$\begin{aligned}
 \text{Snell's Law} & : n_1 \cdot \sin[\theta_1] = n_2 \cdot \sin[\theta_2] \\
 n_1 & = \frac{4}{3}; \quad n_2 = \frac{3}{2}; \quad \theta_1 = 45^\circ = \frac{\pi}{4} \\
 \theta_2 & = \sin^{-1} \left[\frac{n_1}{n_2} \cdot \sin[\theta_1] \right] = \sin^{-1} \left[\left(\frac{\frac{4}{3}}{\frac{3}{2}} \right) \cdot \sin \left[\frac{\pi}{4} \right] \right] \simeq 0.67967 \text{ radians} \simeq 38.94^\circ
 \end{aligned}$$



If the beam is reversed and travels “back” the way it came, then we have to exchange the subscripts:

$$\begin{aligned}
 n_1 & = \frac{3}{2}; \quad n_2 = \frac{4}{3}; \quad \theta_1 \simeq 38.94^\circ \\
 \theta_2 & = \sin^{-1} \left[\frac{n_1}{n_2} \cdot \sin[\theta_1] \right] = \sin^{-1} \left[\left(\frac{\frac{3}{2}}{\frac{4}{3}} \right) \cdot \sin[38.94^\circ] \right] \simeq 0.78533 \text{ radians} = \frac{\pi}{4} \text{ radians} = 45^\circ
 \end{aligned}$$

So the light travels on the same path in both directions...

2. A laser beam impinges on an air-liquid interface at an angle of 55° (measured relative to the normal to the surface). The refracted ray is observed to be transmitted at 40° . What is the refractive index of the liquid?

$$\begin{aligned}
 \text{Snell's Law} & : n_1 \cdot \sin[\theta_1] = n_2 \cdot \sin[\theta_2] \\
 n_1 & = 1; \quad \theta_1 = 55^\circ; \quad \theta_2 = 40^\circ \\
 n_2 & = \frac{n_1 \cdot \sin[\theta_1]}{\sin[\theta_2]} = \frac{1 \cdot \sin[55^\circ]}{\sin[40^\circ]} \simeq 1.2744
 \end{aligned}$$

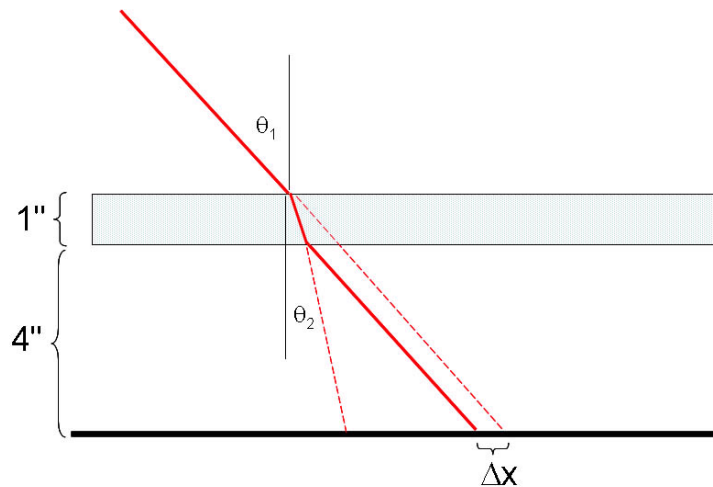
3. A glass plate 1 inch thick with plane-parallel faces and $n = 1.5$ is held horizontally 4 inches above a printed page. If the text is viewed at an angle θ measured relative to the normal to the surface of the glass, find the position of the image of the page formed by rays through the glass. (OPS)

$$\theta_2 = \sin^{-1} \left[\frac{n_1}{n_2} \cdot \sin [\theta_1] \right] = \sin^{-1} \left[\frac{1}{1.5} \cdot \sin [\theta_1] \right]$$

$$\tan [\theta_2] = \frac{\Delta x}{1 \text{ in}} \implies \Delta x = 1 \text{ in} \cdot \tan [\theta_2] = \boxed{25.4 \text{ mm} \cdot \tan \left[\sin^{-1} \left[\frac{1}{1.5} \cdot \sin [\theta_1] \right] \right] = \Delta x}$$

Example : $\theta_1 = 45^\circ \implies \Delta x = 1 \text{ in} \cdot \tan \left[\sin^{-1} \left[\frac{1}{1.5} \cdot \sin [45^\circ] \right] \right] \simeq 0.534 \text{ in} \simeq 13.6 \text{ mm}$

n.b., the 4" measurement does not enter into the solutions



4. An immersed skindiver shines a flashlight at the surface of the water so that the beam makes an angle of 60° relative to the vertical. The refractive index of water is 1.33. (OPS-5.16)

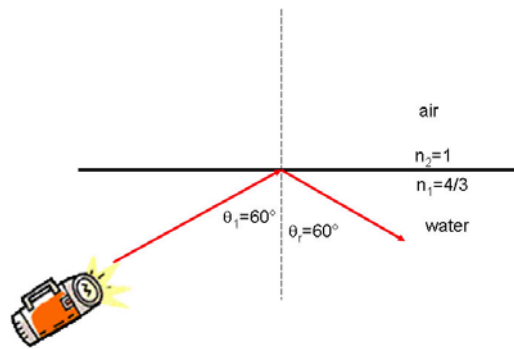
- (a) Where does the flashlight beam go? Assume that there is no transmitted beam of light if there is a reflected one.

From the statement of the problem, the light is either refracted out into the air or totally reflected back into the water. In short, we need to find the critical angle:

$$n_1 \cdot \sin[\theta_c] = n_2 \cdot \sin\left[\frac{\pi}{2}\right]$$

$$\implies \theta_c = \sin^{-1}\left[\frac{n_2}{n_1} \cdot \sin\left[\frac{\pi}{2}\right]\right] = \sin^{-1}\left[\frac{1}{1.33}\right] \simeq 0.85 \text{ radians} \simeq 48.8^\circ$$

Because $\theta_1 > \theta_c$, all of the light is reflected internally



- (b) Oil with index $n = 1.2$ is spread on the water. Now where does the beam of light go?

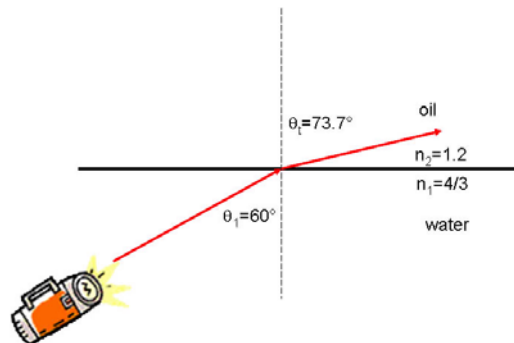
Again, we need to find the critical angle first

$$\theta_c = \sin^{-1}\left[\frac{n_2}{n_1} \cdot \sin\left[\frac{\pi}{2}\right]\right] = \sin^{-1}\left[\frac{1.2}{1.33}\right] \simeq 1.125 \text{ radians} \simeq 64.5^\circ$$

Because $\theta_1 < \theta_c$, all of the light is transmitted into the second medium

The refracted angle is

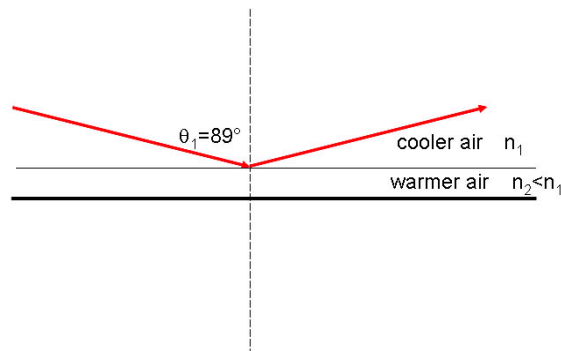
$$\theta_2 = \sin^{-1}\left[\frac{1.33}{1.2} \sin\left[\frac{\pi}{3}\right]\right] \simeq 1.287 \text{ radians} \simeq 73.7^\circ$$



5. The air over a blacktop road is hottest near the surface of the road. The index of air high above the road surface is 1.003. An observer looking at the road from a distance is able to see the pavement only if looking at an angle of 89° or more (measured from the surface normal). Calculate the refractive index of the air at the surface of the road. (OPS-5.16)

The wording may have made this one seem tricky. This is a model of the well-known phenomenon of the “wet road mirage”. The air at the surface of the road is warmer than the air above the road. Warmer air is less dense than cooler air, and thus its refractive index must be lower (it is closer to a vacuum, albeit only a bit closer). Thus we have a “dense-to-rare” reflection, and the critical angle applies.

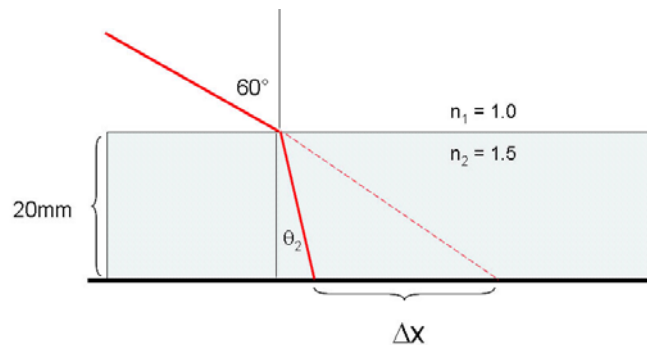
$$\begin{aligned} n_1 \cdot \sin[\theta_c] &= n_2 \cdot \sin\left[\frac{\pi}{2}\right] \\ n_1 &= 1.0003, \theta_c = 89^\circ \\ n_2 &= 1.0003 \cdot \sin[89^\circ] \simeq 1.001 \lesssim 1.003 \end{aligned}$$



6. A ray of light is incident at an angle of 60° on one surface of a glass plate 20 mm thick that is in air. The refractive index of the glass is 1.5. Find the transverse displacement between the incident and emerging rays.

This is almost the same problem as #3

$$\begin{aligned} 1 \cdot \sin[60^\circ] &= 1.5 \cdot \sin[\theta_2] \\ \theta_2 &= \sin^{-1}\left[\frac{1}{1.5} \sin[60^\circ]\right] \simeq 0.61547 \text{ radians} \simeq 35.6^\circ \\ \Delta x &= 20 \text{ mm} \cdot \tan[\theta_2] = 20 \text{ mm} \cdot \tan\left[\sin^{-1}\left[\frac{1}{1.5} \sin[60^\circ]\right]\right] \simeq 14.1 \text{ mm} \end{aligned}$$



7. (OPTIONAL – EXTRA CREDIT) The bore of a cylindrical thermometer (i.e., the hole in the middle for the mercury) has diameter equal to half the outside diameter of the stem. The refractive index of the glass is 1.5.

- (a) Determine the apparent width of the column of mercury relative to the apparent width of the stem, assuming that the viewing distance is large compared to the diameter of the thermometer.

$$1.5 \cdot \sin [\theta_1] = 1.0 \cdot \sin [\theta_2]$$

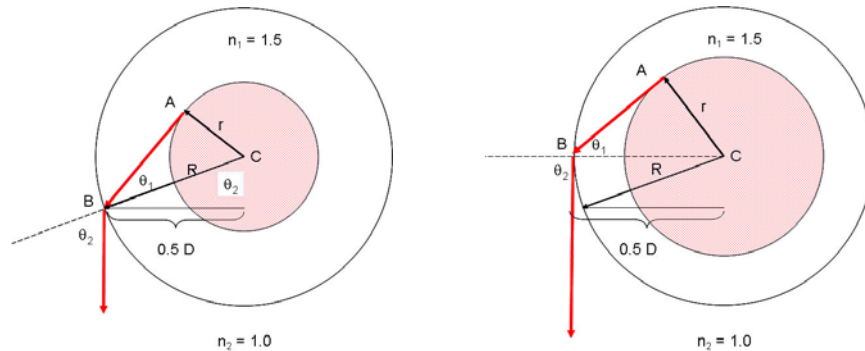
ΔABC is a right triangle, the right angle is located at B

$$\text{if } \frac{R}{r} = 2, \text{ then angle } B \text{ is } 30^\circ$$

$$1.5 \cdot \sin \left[\frac{\pi}{6} \right] = 1.0 \cdot \sin [\theta_2]$$

$$\theta_2 = \sin^{-1} \left[\frac{3}{2} \cdot \frac{1}{2} \right] = \sin^{-1} \left[\frac{3}{4} \right] \simeq 0.85 \text{ radians}$$

$$\text{Apparent width} = 2 \cdot R \sin [\theta_r] = 2R \cdot \frac{3}{4} = D \cdot \frac{3}{4}$$



- (b) Determine the apparent width if the diameter of the bore is $2/3$ the diameter of the stem? (HINT: draw the picture)

Same picture:

$$1.5 \cdot \sin [\theta_1] = 1.0 \cdot \sin [\theta_2]$$

ΔABC is a right triangle, the right angle is located at B

$$\frac{R}{r} = \frac{3}{2} \implies \sin [\theta_1] = \frac{2}{3}$$

$$\frac{3}{2} \cdot \sin [\theta_1] = \frac{3}{2} \cdot \frac{2}{3} = 1 = 1.0 \cdot \sin [\theta_2]$$

$$\theta_2 = \sin^{-1} [1] = \frac{\pi}{2} \text{ radians} = 90^\circ$$

$$\text{Apparent width} = 2 \cdot R = D$$

The apparent diameter of the mercury column is equal to the diameter of the thermometer