SIMG-303-20033 Solution Set #1

Solve the following problems. You may "plot" any results requested by computer, BUT include the graphs with the problem, NOT as a cluster of printouts at the beginning or end of the assignment.

1. A source of harmonic motion of the form $y(t) = 6 \cdot \cos(\omega_0 t)$ located at the origin of the spatial coordinate system emits a wave that travels through a uniform (i.e., homogeneous) medium at a rate of 60 mm per second.

   (a) Find the formula for the displacement due to this wave at a distance of 800 mm from the origin.

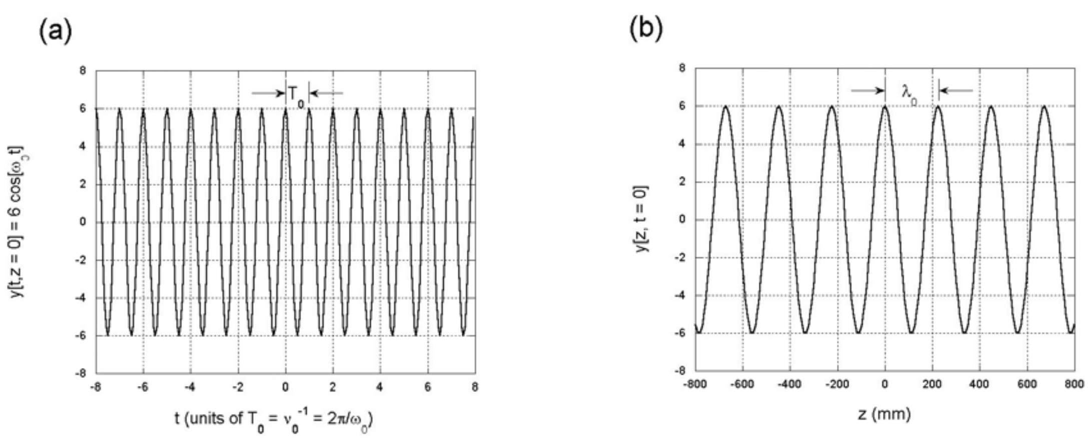
   This is a "travelling wave" which has a sinusoidal form in both time AND space. The spatial part of the wave at the origin must have the same functional form as $y[t]$*:*

   $$y[z=0,t] = 6 \cdot \cos(\omega_0 t) = 6 \cdot \cos(2\pi\nu_0 t)$$

   where $\nu_0 = \frac{\omega_0}{2\pi}$*. The displacement of the function that was located at the origin at* $t = 0$ *will travel to the point located* 60 mm *away at* $t = 1\,\mathrm{s}$*. The amplitude will travel to the point located* 800 mm *away in* $t = \frac{800\,\mathrm{mm}}{60\,\frac{\mathrm{mm}}{\mathrm{s}}} = \frac{40}{3}\,\mathrm{s}$*. Consult the sketch (a) to see the amplitude evaluated as a function of time at the origin* ($z = 0$)*. Sketch (b) shows the amplitude as a function of position* z *at* $t = 0\,\mathrm{s}$*, i.e., it is a "snapshot" of the spatial wave at this one time. Note that this wave has a "spatial period" labeled by* λ_0*, the wavelength of the spatial oscillation. The amplitude (displacement) of the wave at the distance* $z = 800\,\mathrm{mm}$ *is:*

   $$y[z = 800\,\mathrm{mm}, t] = 6 \cdot \cos\left(2\pi\nu_0 t - 2\pi\frac{800\,\mathrm{mm}}{\lambda_0\,[\mathrm{mm}]}\right)$$

   where λ_0 *is not specified.*



We do know the relationship for the velocity, wavelength, and temporal frequency:

$$\begin{aligned}
\mathrm{v}_0 &= \lambda_0 \cdot \nu_0 \Longrightarrow \lambda_0 = \frac{\mathrm{v}_0}{\nu_0} \\
&\Longrightarrow y\,[z,t] = A_0 \cdot \cos\left(2\pi\nu_0 t - 2\pi\nu_0 \frac{z}{\mathrm{v}_0}\right) \\
y\,[z,t] &= 6 \cdot \cos\left(\omega_0 \left(t - \frac{800\,\mathrm{mm}}{\nu_0\lambda_0\left[\frac{\mathrm{mm}}{\mathrm{s}}\right]}\right)\right) \\
&= 6 \cdot \cos\left(\omega_0 \left(t - \frac{800\,\mathrm{mm}}{\mathrm{v}_0\left[\frac{\mathrm{mm}}{\mathrm{s}}\right]}\right)\right) \\
&= 6 \cdot \cos\left(\omega_0 \left(t - \frac{800\,\mathrm{mm}}{60\frac{\mathrm{mm}}{\mathrm{s}}}\right)\right) \\
y\,[z = 800\,\mathrm{mm}, t] &= 6 \cdot \cos\left(\omega_0 \left(\frac{800\,\mathrm{mm}}{60\frac{\mathrm{mm}}{\mathrm{s}}} - t\right)\right) \\
&= 6 \cdot \cos\left(\omega_0 \left(\frac{800}{60}\,\mathrm{s} - t\right)\right)
\end{aligned}$$

(b) Find the displacement at that distance for $t = 60\,\mathrm{s}$.
Just substitute this time into the equation in part (a):

$$y\,[z = 800\,\mathrm{mm}, t = 60\,\mathrm{s}] = 6 \cdot \cos\left(\omega_0 \left(\frac{800\,\mathrm{mm}}{60\frac{\mathrm{mm}}{\mathrm{s}}} - 60\,\mathrm{s}\right)\right)$$

2. Find expressions for the sums of the following pairs of waves AND sketch the sum (you may "plot" the result by computer if you wish), i.e., for $f_1[x]$ and $f_2[x]$ in each case, find $g[x] = f_1[x] + f_2[x]$.

The general expression for the sum of two sinusoidal waves WITH THE SAME AMPLITUDE is obtained from the Euler relation:

$$\begin{aligned}
&\exp[+i(\omega_1 t + \phi_1)] + \exp[+i(\omega_2 t + \phi_2)] \\
= &\exp\left[+i\left(\frac{\omega_1}{2}t + \frac{\phi_1}{2}\right)\right] \cdot \exp\left[+i\left(\frac{\omega_1}{2}t + \frac{\phi_1}{2}\right)\right] \\
&+\exp\left[+i\left(\frac{\omega_2}{2}t + \frac{\phi_2}{2}\right)\right] \cdot \exp\left[+i\left(\frac{\omega_2}{2}t + \frac{\phi_2}{2}\right)\right] \\
= &\exp\left[+i\left(\frac{\omega_1}{2}t + \frac{\phi_1}{2}\right)\right] \cdot \exp\left[+i\left(\frac{\omega_1}{2}t + \frac{\phi_1}{2}\right)\right] \cdot 1 \\
&+\exp\left[+i\left(\frac{\omega_2}{2}t + \frac{\phi_2}{2}\right)\right] \cdot \exp\left[+i\left(\frac{\omega_2}{2}t + \frac{\phi_2}{2}\right)\right] \cdot 1
\end{aligned}$$

where we write the "1" as the product of cancelling exponents

$$\begin{aligned}
&\exp[+i(\omega_1 t + \phi_1)] + \exp[+i(\omega_2 t + \phi_2)] \\
= &\exp\left[+i\left(\frac{\omega_1}{2}t + \frac{\phi_1}{2}\right)\right] \cdot \exp\left[+i\left(\frac{\omega_1}{2}t + \frac{\phi_1}{2}\right)\right] \cdot \exp\left[+i\left(\frac{\omega_2}{2}t + \frac{\phi_2}{2}\right)\right] \cdot \exp\left[-i\left(\frac{\omega_2}{2}t + \frac{\phi_2}{2}\right)\right] \\
&+\exp\left[+i\left(\frac{\omega_1}{2}t + \frac{\phi_1}{2}\right)\right] \cdot \exp\left[-i\left(\frac{\omega_1}{2}t + \frac{\phi_1}{2}\right)\right] \cdot \exp\left[+i\left(\frac{\omega_2}{2}t + \frac{\phi_2}{2}\right)\right] \cdot \exp\left[+i\left(\frac{\omega_2}{2}t + \frac{\phi_2}{2}\right)\right]
\end{aligned}$$

$$\begin{aligned}
&\exp[+i(\omega_1 t + \phi_1)] + \exp[+i(\omega_2 t + \phi_2)] \\
= &\left(e^{+i\left(\frac{\omega_1}{2}t+\frac{\phi_1}{2}\right)} \cdot e^{+i\left(\frac{\omega_1}{2}t+\frac{\phi_1}{2}\right)}\right)\left(e^{+i\left(\frac{\omega_2}{2}t+\frac{\phi_2}{2}\right)} \cdot e^{-i\left(\frac{\omega_2}{2}t+\frac{\phi_2}{2}\right)}\right) \\
&+\left(e^{+i\left(\frac{\omega_1}{2}t+\frac{\phi_1}{2}\right)} \cdot e^{-i\left(\frac{\omega_1}{2}t+\frac{\phi_1}{2}\right)}\right)\left(e^{+i\left(\frac{\omega_2}{2}t+\frac{\phi_2}{2}\right)} \cdot e^{+i\left(\frac{\omega_2}{2}t+\frac{\phi_2}{2}\right)}\right) \\
= &\left(e^{+i\left(\frac{\omega_1}{2}t+\frac{\phi_1}{2}\right)} \cdot e^{+i\left(\frac{\omega_2}{2}t+\frac{\phi_2}{2}\right)}\right)\left(e^{+i\left(\frac{\omega_1}{2}t+\frac{\phi_1}{2}\right)} \cdot e^{-i\left(\frac{\omega_2}{2}t+\frac{\phi_2}{2}\right)}\right) \\
&+\left(e^{+i\left(\frac{\omega_1}{2}t+\frac{\phi_1}{2}\right)} \cdot e^{+i\left(\frac{\omega_2}{2}t+\frac{\phi_2}{2}\right)}\right)\left(e^{-i\left(\frac{\omega_1}{2}t+\frac{\phi_1}{2}\right)} \cdot e^{+i\left(\frac{\omega_2}{2}t+\frac{\phi_2}{2}\right)}\right) \\
= &\left(e^{+i\left(\frac{\omega_1}{2}t+\frac{\phi_1}{2}\right)} \cdot e^{+i\left(\frac{\omega_2}{2}t+\frac{\phi_2}{2}\right)}\right)\left[\left(e^{+i\left(\frac{\omega_1}{2}t+\frac{\phi_1}{2}\right)} \cdot e^{-i\left(\frac{\omega_2}{2}t+\frac{\phi_2}{2}\right)}\right) + \left(e^{-i\left(\frac{\omega_1}{2}t+\frac{\phi_1}{2}\right)} \cdot e^{+i\left(\frac{\omega_2}{2}t+\frac{\phi_2}{2}\right)}\right)\right] \\
= &\left(e^{+i\left(\frac{\omega_1}{2}t+\frac{\phi_1}{2}\right)} \cdot e^{+i\left(\frac{\omega_2}{2}t+\frac{\phi_2}{2}\right)}\right)\left[e^{+i\left(\frac{\omega_1-\omega_2}{2}t+\frac{\phi_1-\phi_2}{2}\right)} + e^{-i\left(\frac{\omega_1-\omega_2}{2}t+\frac{\phi_1-\phi_2}{2}\right)}\right] \\
= &\left(e^{+i\left(\frac{\omega_1+\omega_2}{2}t+\frac{\phi_1+\phi_2}{2}\right)}\right) \cdot 2\cos\left[\left(\frac{\omega_1-\omega_2}{2}\right)t + \left(\frac{\phi_1-\phi_2}{2}\right)\right]
\end{aligned}$$

Take the real part of both sides:

$$\begin{aligned}
&\mathrm{Re}\left\{\exp[+i(\omega_1 t + \phi_1)] + \exp[+i(\omega_2 t + \phi_2)]\right\} \\
= &\mathrm{Re}\left\{\left(e^{+i\left(\frac{\omega_1+\omega_2}{2}t+\frac{\phi_1+\phi_2}{2}\right)}\right) \cdot 2\cos\left[\left(\frac{\omega_1-\omega_2}{2}\right)t + \left(\frac{\phi_1-\phi_2}{2}\right)\right]\right\} \\
&\cos[\omega_1 t + \phi_1] + \cos[\omega_2 t + \phi_2] \\
= &2\cos\left[\left(\frac{\omega_1-\omega_2}{2}\right)t + \left(\frac{\phi_1-\phi_2}{2}\right)\right] \cdot \cos\left[\left(\frac{\omega_1+\omega_2}{2}\right)t + \left(\frac{\phi_1+\phi_2}{2}\right)\right]
\end{aligned}$$

In words, the sum of two equal-amplitude sinusoidal waves is equivalent to the product of two sinusoidal waves: one with the average phase and one with half of the difference of the phase (the so-called "modulation phase")

(a) $f_1[x] = 2 \cdot \cos[2\pi\xi_0 x]\,,\; f_2[x] = 2 \cdot \cos[2\pi\xi_0 x]$

$$\begin{aligned} \textit{By simple sum} &: \\ f_1[x] + f_2[x] &= (2 \cdot \cos[2\pi\xi_0 x]) + (2 \cdot \cos[2\pi\xi_0 x]) \\ &= 4 \cdot (\cos[2\pi\xi_0 x] + \cos[2\pi\xi_0 x]) \\ &= 8 \cdot \cos[2\pi\xi_0 x] \end{aligned}$$

$$\begin{aligned} \textit{From the formula} &: \\ f_1[x] + f_2[x] &= (2 \cdot \cos[2\pi\xi_0 x]) + (2 \cdot \cos[2\pi\xi_0 x]) \\ &= 2 \cdot (\cos[2\pi\xi_0 x] + \cos[2\pi\xi_0 x]) \\ &= 2 \cdot 2 \cdot \cos\left[2\pi\left(\frac{\xi_0 + \xi_0}{2}\right)x\right] \cdot \cos\left[2\pi\left(\frac{\xi_0 - \xi_0}{2}\right)x\right] \\ &= 4 \cdot \cos[2\pi(\xi_0)x] \cdot \cos[2\pi(0)x] \\ &= 4 \cdot \cos[2\pi(\xi_0)x] \cdot 1 \end{aligned}$$

(b) $f_1[x] = 2 \cdot \cos[2\pi\xi_0 x]\,,\; f_2[x] = \cos[2\pi\xi_0 x]$

$$\begin{aligned} \textit{By simple sum} &: \\ f_1[x] + f_2[x] &= (2 \cdot \cos[2\pi\xi_0 x]) + (1 \cdot \cos[2\pi\xi_0 x]) \\ &= 2 \cdot (\cos[2\pi\xi_0 x] + \cos[2\pi\xi_0 x]) \\ &= 3 \cdot \cos[2\pi\xi_0 x] \end{aligned}$$

(c) $f_1[x] = 2 \cdot \cos[2\pi\xi_0 x]\,,\; f_2[x] = \cos\left[2\pi\xi_0 x + \frac{\pi}{2}\right]$
Can't do this by simple sum...

From the formula:

$$f_1[x] + f_2[x] = (2 \cdot \cos[2\pi\xi_0 x]) + \cos\left[2\pi\xi_0 x + \frac{\pi}{2}\right]$$

$$= \cos[2\pi\xi_0 x] + \left(\cos[2\pi\xi_0 x] + \cos\left[2\pi\xi_0 x + \frac{\pi}{2}\right]\right)$$

$$= \cos[2\pi\xi_0 x] + 2 \cdot \cos\left[2\pi\left(\frac{\xi_0 + \xi_0}{2}\right)x + \frac{0 + \frac{\pi}{2}}{2}\right] \cdot \cos\left[2\pi\left(\frac{\xi_0 - \xi_0}{2}\right)x + \frac{0 - \frac{\pi}{2}}{2}\right]$$

$$= \cos[2\pi\xi_0 x] + 2 \cdot \cos\left[2\pi\xi_0 x + \frac{\pi}{4}\right] \cdot \cos\left[0 - \frac{\pi}{4}\right]$$

$$= \cos[2\pi\xi_0 x] + \left(2 \cdot \cos\left[+\frac{\pi}{4}\right]\right) \cdot \cos\left[2\pi\xi_0 x + \frac{\pi}{4}\right]$$

(d) $f_1[x] = \cos[2\pi\xi_0 x]$, $f_2[x] = \cos\left[2\pi\xi_0 x - \frac{\pi}{4}\right]$

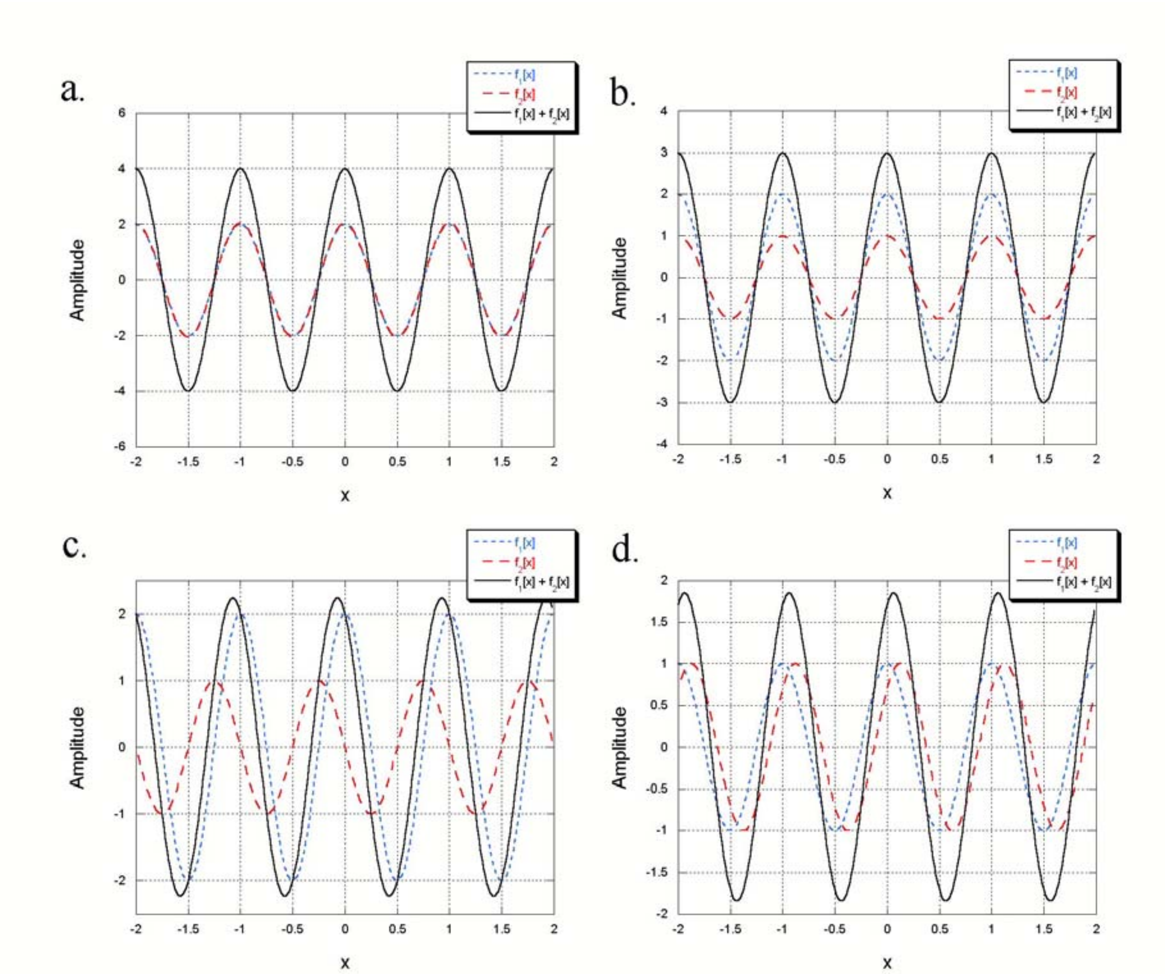
From the formula:

$$f_1[x] + f_2[x] = (\cos[2\pi\xi_0 x]) + \cos\left[2\pi\xi_0 x - \frac{\pi}{4}\right]$$

$$= 2 \cdot \cos\left[2\pi\left(\frac{\xi_0 + \xi_0}{2}\right)x + \frac{0 + \left(-\frac{\pi}{4}\right)}{2}\right] \cdot \cos\left[2\pi\left(\frac{\xi_0 - \xi_0}{2}\right)x + \frac{0 - \left(-\frac{\pi}{4}\right)}{2}\right]$$

$$= 2 \cdot \cos\left[2\pi\xi_0 x - \frac{\pi}{8}\right] \cdot \cos\left[2\pi(0)x + \frac{\pi}{8}\right]$$

$$= \left(2 \cdot \cos\left[+\frac{\pi}{8}\right]\right) \cdot \cos\left[2\pi\xi_0 x - \frac{\pi}{8}\right]$$



3. Use the Euler relation: $\exp[i\theta] = \cos[\theta] + i\ \sin[\theta]$ to derive expressions for the following in terms of $\cos[\theta_1]$, $\sin[\theta_1]$, $\cos[\theta_2]$, and $\sin[\theta_2]$:

   (a) $\cos[2\theta_1]$

   This one is straightforward:

$$\begin{aligned}
\exp[i(2\theta)] &= (\exp[i\theta])^2 = (\cos[\theta] + i\ \sin[\theta])^2 \\
&= \cos^2[\theta] + i(2\cos[\theta]\cdot\sin[\theta]) + (i\sin[\theta])^2 \\
&= \left(\cos^2[\theta] - \sin^2[\theta]\right) + i(2\cos[\theta]\cdot\sin[\theta]) \\
\exp[i(2\theta)] &= \cos[2\theta] + i\ \sin[2\theta] \\
\textit{Equate real and imaginary parts} &: \\
\operatorname{Re}\{\exp[i(2\theta)]\} &= \cos[2\theta] = \cos^2[\theta] - \sin^2[\theta] \\
\operatorname{Im}\{\exp[i(2\theta)]\} &= \sin[2\theta] = 2\cos[\theta]\cdot\sin[\theta]
\end{aligned}$$

   (b) $\cos[\theta_1]\cdot\cos[\theta_2]$

   This one is trickier:

$$\cos[\theta_1] = \frac{1}{2}\exp[+i\theta_1] + \frac{1}{2}\exp[-i\theta_1]$$

$$\cos[\theta_2] = \frac{1}{2}\exp[+i\theta_2] + \frac{1}{2}\exp[-i\theta_2]$$

$$\cos[\theta_1]\cdot\cos[\theta_2] = \left(\frac{1}{2}\exp[+i\theta_1] + \frac{1}{2}\exp[-i\theta_1]\right)\cdot\left(\frac{1}{2}\exp[+i\theta_2] + \frac{1}{2}\exp[-i\theta_2]\right)$$

$$= \frac{1}{4}\left(\exp[+i(\theta_1+\theta_2)] + \frac{1}{2}\exp[-i(\theta_1+\theta_2)] + \exp[+i(\theta_1-\theta_2)] + \exp[-i(\theta_1-\theta_2)]\right)$$

$$= \frac{1}{2}\cdot\left(\frac{1}{2}(\exp[+i(\theta_1+\theta_2)] + \exp[-i(\theta_1+\theta_2)]) + \frac{1}{2}(\exp[+i(\theta_1-\theta_2)] + \exp[-i(\theta_1-\theta_2)])\right)$$

$$= \frac{1}{2}\cdot(\cos[\theta_1+\theta_2] + \cos[\theta_1-\theta_2])$$

   (c) $\sin[\theta_1-\theta_2]$

$$\begin{aligned}
\sin[\theta_1-\theta_2] &= \operatorname{Im}\{\exp[i(\theta_1-\theta_2)]\} \\
&= \operatorname{Im}\{\exp[i\theta_1]\cdot\exp[-i\theta_2]\} \\
&= \operatorname{Im}\{(\cos[\theta_1] + i\sin[\theta_1])\cdot(\cos[\theta_2] - i\sin[\theta_2])\} \\
&= \operatorname{Im}\{\cos[\theta_1]\cdot\cos[\theta_2] + \sin[\theta_1]\cdot\sin[\theta_2] + i\sin[\theta_1]\cdot\cos[\theta_2] - i\sin[\theta_2]\cdot\cos[\theta_1]\} \\
&= \sin[\theta_1]\cdot\cos[\theta_2] - \sin[\theta_2]\cdot\cos[\theta_1]
\end{aligned}$$

4. For each of the following complex numbers z, plot the location of z on the complex plane (i.e., plot the Argand diagram), plot the complex conjugate z^*, z^{-1}, $z+z^*$ and $z-z^*$

(a) $z = 1+2i$

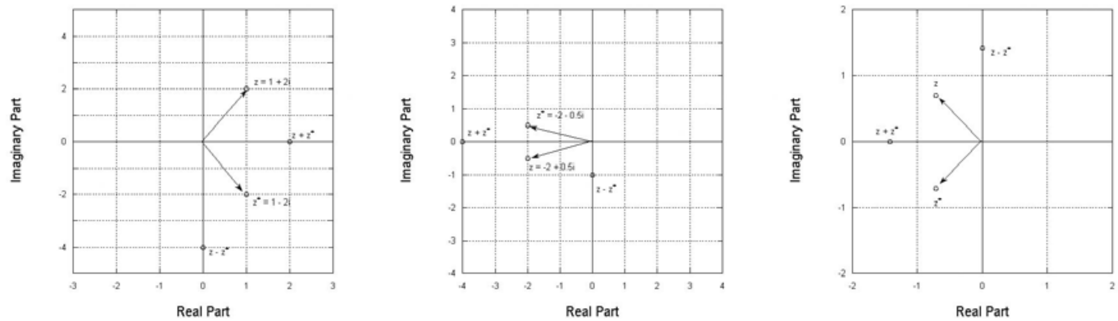
$$\begin{aligned}
z &= 1+2i \\
z^* &= 1+2(-i) = 1-2i \\
z^{-1} &= \frac{1}{1+2i} = \frac{1}{1+2i}\cdot\frac{1-2i}{1-2i} = \frac{1-2i}{1+4} = \frac{1}{5}+i\left(-\frac{2}{5}\right) \\
z+z^* &= (1+2i)+(1-2i) = (1+1)+i(2-2) = +2 = 2\cdot\mathrm{Re}\{z\} \\
z-z^* &= (1+2i)-(1-2i) = (1+1)+i(2-(-2)) = +4i = i(2\cdot\mathrm{Im}\{z\})
\end{aligned}$$

(b) $z = \frac{1-4i}{2i}$

$$\begin{aligned}
z &= \frac{1-4i}{2i} = \frac{1}{2i}-\frac{4i}{2i} = \frac{1}{2i}-2 = -2+\frac{1}{2i}\cdot\frac{-i}{-i} = -2+i\left(-\frac{1}{2}\right) \\
z^* &= -2-i\left(-\frac{1}{2}\right) = -2+i\left(+\frac{1}{2}\right) \\
z^{-1} &= \frac{1}{-2+i\left(-\frac{1}{2}\right)} = \frac{-2+i\left(\frac{1}{2}\right)}{4+\frac{1}{4}} = \frac{-8}{17}+i\frac{2}{17} \\
z+z^* &= \left(-2+i\left(-\frac{1}{2}\right)\right)+\left(-2+i\left(+\frac{1}{2}\right)\right) = -4 \\
z+z^* &= \left(-2+i\left(-\frac{1}{2}\right)\right)-\left(-2+i\left(+\frac{1}{2}\right)\right) = -i
\end{aligned}$$

(c) $z = 2\cdot\exp\left[i\cdot\frac{3\pi}{4}\right]$

$$\begin{aligned}
z &= 2\cdot\exp\left[i\cdot\frac{3\pi}{4}\right] = 2\cdot\left(\cos\left[\frac{3\pi}{4}\right]+i\sin\left[\frac{3\pi}{4}\right]\right) \\
&= (-1+i)\sqrt{2} = \sqrt{2}(-1+i) \\
z^* &= \sqrt{2}(-1-i) \\
z^{-1} &= \frac{1}{2\cdot\exp\left[i\cdot\frac{3\pi}{4}\right]} = \frac{1}{2}\exp\left[-i\cdot\frac{3\pi}{4}\right] \\
z+z^* &= \sqrt{2}(-1+i)+\sqrt{2}(-1-i) = -2\sqrt{2} \\
z-z^* &= \sqrt{2}(-1+i)-\sqrt{2}(-1-i) = 0+i\left(2\sqrt{2}\right)
\end{aligned}$$


z = 1 + 2i
z + z*
z* = 1 - 2i
z - z*
Imaginary Part
Real Part
z* = -2 - 0.5i
z + z*
z = -2 + 0.5i
z - z*
Imaginary Part
Real Part
z - z*
z
z + z*
z*
Imaginary Part
Real Part

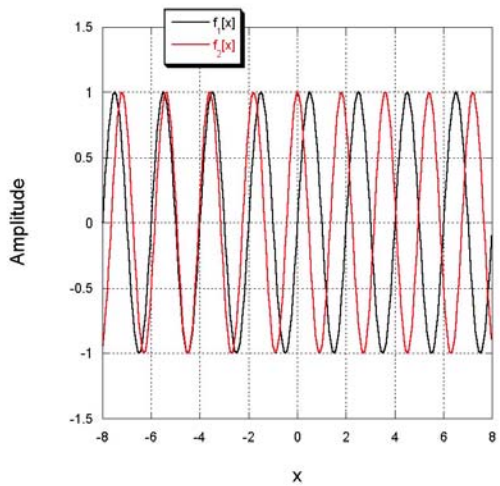
5. Consider two oscillations at different frequencies:

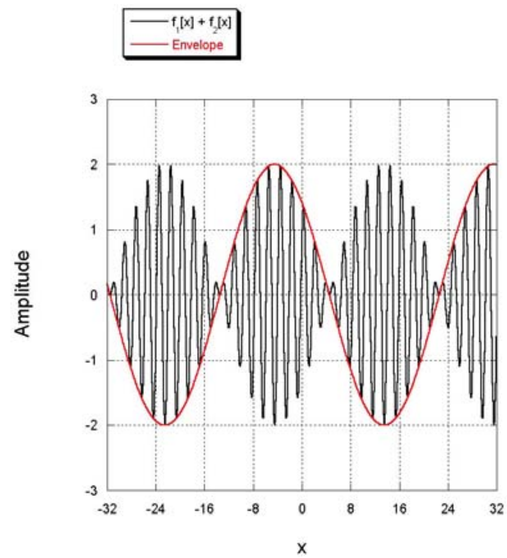
$$f_1\left[x\right] = \cos\left[2\pi\frac{x}{2} - \frac{\pi}{2}\right] = \sin\left[2\pi\frac{x}{2}\right]$$

$$f_2\left[x\right] = \cos\left[2\pi\frac{x}{1.8}\right]$$

(a) Sketch (or plot) the two oscillations separately (preferably on the same graph):

(b) Sketch (or plot) the sum of the two oscillations over a sufficiently large region of the real axis to get a good idea of the behavior





Note difference between domains! Individual sinusoids are plotted for $-8 \leq x \leq +8$, while the sum is plotted for $-32 \leq x \leq +32$.

(c) Find an expression for this sum as the product of two different sinusoidal oscillations.

$$\begin{aligned}
f_1\left[x\right] + f_2\left[x\right] &= \cos\left[2\pi\frac{x}{2} - \frac{\pi}{2}\right] + \cos\left[2\pi\frac{x}{1.8}\right] \\
&= 2\cdot\cos\left[2\pi x\left(\frac{\frac{1}{2}+\frac{1}{1.8}}{2}\right) + \left(\frac{-\frac{\pi}{2}+0}{2}\right)\right]\cdot\cos\left[2\pi x\left(\frac{\frac{1}{2}-\frac{1}{1.8}}{2}\right) + \left(\frac{-\frac{\pi}{2}-0}{2}\right)\right] \\
&= 2\cdot\cos\left[2\pi x\left(\frac{\frac{1}{2}+\frac{1}{1.8}}{2}\right) + \left(\frac{-\frac{\pi}{2}+0}{2}\right)\right]\cdot\cos\left[2\pi x\left(\frac{\frac{1}{2}-\frac{1}{1.8}}{2}\right) + \left(\frac{-\frac{\pi}{2}-0}{2}\right)\right] \\
&= 2\cdot\cos\left[2\pi x\left(\frac{\frac{1}{2}+\frac{1}{1.8}}{2}\right) - \frac{\pi}{4}\right]\cdot\cos\left[2\pi x\left(\frac{\frac{1}{2}-\frac{1}{1.8}}{2}\right) - \frac{\pi}{4}\right] \\
&\simeq 2\cdot\cos\left[2\pi\left(\frac{x}{1.8947}\right) - \frac{\pi}{4}\right]\cdot\cos\left[2\pi\left(\frac{x}{-36}\right) - \frac{\pi}{4}\right] \\
&\quad 2\cdot\cos\left[2\pi\left(\frac{x}{1.8947}\right) - \frac{\pi}{4}\right]\cdot\cos\left[2\pi\left(\frac{x}{36}\right) + \frac{\pi}{4}\right]
\end{aligned}$$

The second term is slowly varying and appears as the "envelope" of the oscillation.