

Select **FIVE (5)** of the following problems (equal weighting).

Staple problems together and submit *in numerical order*.

Closed Book, No Notes

Calculators with no programmed equations are allowed, but not necessary.

1. Define the following terms IN WORDS; you may use equations to illustrate your definition, but I'm more interested in the verbal description:

- (a) oscillation (include descriptions of the two necessary forces)
- (b) traveling wave
- (c) index of refraction
- (d) dispersion relation
- (e) phase velocity
- (f) modulation velocity (also called "group velocity")

2. Given the following complex numbers z_n , evaluate the real part, imaginary part, magnitude, phase angle, z^{-1} and z^* and locate them on an Argand diagram. You may leave quantities in the form of functions – you are not required to evaluate numerical values

- (a) $z_1 = 4 \cdot \exp\left[-\frac{3\pi}{4}\right]$

- (b) $z_2 = \frac{6i+3}{3i}$

- (c) $z_3 = \sin\left[\frac{\pi t}{2}\right] + i \cos\left[\frac{\pi t}{2}\right]$

3. Two traveling waves have slightly different temporal frequencies ν_1 and $\nu_2 = \nu_1 + \Delta\nu$, and thus slightly different wavelengths λ_1 and $\lambda_2 = \lambda_1 + \Delta\lambda$. Demonstrate the validity of the expression:

$$\left|\frac{\Delta\lambda}{\lambda}\right| \simeq \left|\frac{\Delta\nu}{\nu}\right|$$

4. The dispersion relation for sinusoidal waves in the ionosphere is:

$$\omega^2 = \omega_p^2 + c^2 k^2$$

where ω_p is a constant, the so-called *plasma frequency*. A typical approximate value for ω_p is:

$$\omega_p = 2\pi\nu_p \simeq 0.125 \times 10^9 \frac{\text{radians}}{\text{sec}}$$

- (a) Find expressions for the phase velocity v_ϕ and the modulation (or group) velocity v_{mod} . (HINT: evaluate total derivatives of both sides).

- (b) Calculate $v_\phi \cdot v_{\text{mod}}$

MORE→→→→

5. (Read carefully) The phase velocity of the waves in some medium is proportional to ω^{-2}
- (a) Sketch the dispersion relation $\omega [k]$ in this medium.
 - (b) Find an expression for the modulation velocity
 - (c) Do the waves exhibit normal or anomalous dispersion?

6. Consider the superposition of two sinusoidal traveling waves:

$$\begin{aligned}f_1 [z, t] &= A_1 \sin [k_1 z - \omega_1 t] \\f_2 [z, t] &= -A_2 \sin [k_2 z - \omega_2 t]\end{aligned}$$

where $A_1 = A_2 = 10 \text{ mm}$, $\nu_1 = 100 \text{ Hz}$, $v_1 = 2.5 \frac{\text{m}}{\text{s}}$, $\nu_2 = 150 \text{ Hz}$, $v_2 = 5.0 \frac{\text{m}}{\text{s}}$

- (a) Find the resulting wave in terms of the average wave and the modulation wave.
 - (b) Calculate the wavelengths of the two waves f_1 and f_2 .
 - (c) Find the velocity of the average wave and of the modulation wave.
 - (d) Does this system exhibit normal or anomalous dispersion?
7. We derived an expression for the superposition (sum) of multiple oscillations with the same frequency. Convince me with words and diagrams that this result is correct.