Chapter 8

Optical Imaging

8.1 Transition from Wave Optics to Ray Optics

We have mentioned that the rigorous evaluation of light upon interaction with matter (as in diffracting apertures, mirrors, or lenses) involves the solution of the four equations collected by Maxwell subject to the specific conditions of the problem. The exact solution to these equations is often very difficult to obtain. Fortunately, it is often sufficient to find approximate solutions. The very useful approximation relevant to imaging applications is the model of light as *rays*, which is generally called *geometrical optics*. This approximation emphasizes the path travelled by light through media to find the locations and sizes of images. The other model of optics as waves, called *physical optics*, emphasizes the "deviation" or "spreading" of light from the geometrical paths to create interference and/or diffraction, and demonstrates the fundamental limitations on the performance (*i.e.,* the resolution) of optical imaging systems.

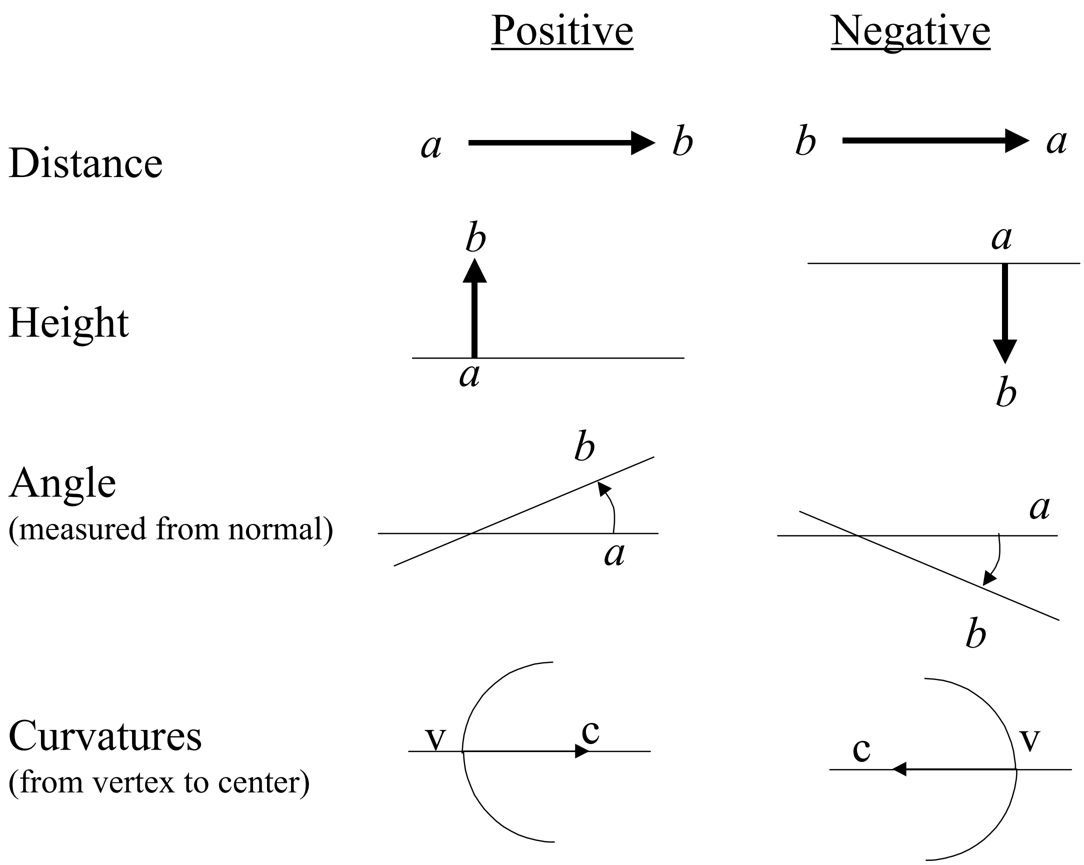
> Ray: a line in space that maps the direction of energy flow. It is a mathematical construction, not an actual entity.

The geometrical optics approximation is a limiting case of the more general wave optical model in the limit that the wavelength λ of light goes to zero. As we shall see later, there is no diffraction in the wave model in this case.

8.1.1 Notational Conventions

One of the more confusing and frustrating aspects of geometrical optics is the existence of multiple notational conventions. These notes use the convention of *directed distances*, which is also used by Halliday and Resnik, Jenkins and White, Hecht, Nussbaum and Phillips, Crawford, Iizuka, Goodman, and Gaskill. The other common convention us based on a coordinate system with the origin at the first vertex of the optical system, and is used by many authors for lens design, e.g., Born and Wolf, Warren Smith, and Ditchburn.

A powerful advantage of the directed distance convention is the resulting (and pleasing) symmetry between objects and images, and because the nature of the the resulting images is obvious.



1. Light travels from left to right;
2. Interfaces (*i.e.,* lens or mirror surfaces) are numbered from the first to the last encountered by the ray;
3. Distances are measured from lens *vertices*, the intersection of the lens surface with the axis of symmetry (optical axis);
4. A horizontal distance is positive if measured from left to right;
5. A vertical distance from the axis is positive if measured “up”;
6. Angles are positive if measured from the optical axis in the counterclockwise direction;
7. A radius of curvature is positive if the center is to the right of the vertex;
8. A subscript on a quantity corresponds to the surface with which is it associated;
9. If used, primed quantities (*e.g.,* n') refer to the “outgoing” side of an interface. These are useful when describing a multiple element system where the output (image) space for one element is the input (object) space for the next element.

8.1.2 Fermat’s Principle

Hero of Alexandria hypothesized the model of light propagation that could be called the *principle of least distance*:

A ray of light traveling between two arbitrary points
traverses the shortest possible path in space.

This statement applies to reflection and transmission through homogeneous media (i.e., the media is characterized by a single index of refraction). **However**, it dies not work when the object and observation points are in two different media (*i.e.,* for refraction) or if multiple media are present between the points.

In 1657, Pierre Fermat modified Hero's statement to formulate the *principle of least time:*

A light ray travels the path that requires the least time.

The laws of reflection and refraction may be easily derived from Fermat's principle. A moving ray (or car, bullet, or baseball) traveling a distance s at a velocity v requires t seconds:

$$t = \frac{s}{\mathrm{v}}$$

If the ray travels at different velocities for different increments of distance, the travel time may be written as:

$$t = \sum_{m=1}^{M} \frac{s_m}{\mathrm{v}_m}$$

We know that the velocity of a light ray in a medium of index n is $\mathrm{v} = \frac{c}{n}$, so that:

$$t = \sum_{m=1}^{M} \frac{s_m}{\left(\frac{c}{n_m}\right)} = \frac{1}{c}\sum_{m=1}^{M}(n_m s_m) = \frac{\ell}{c}$$

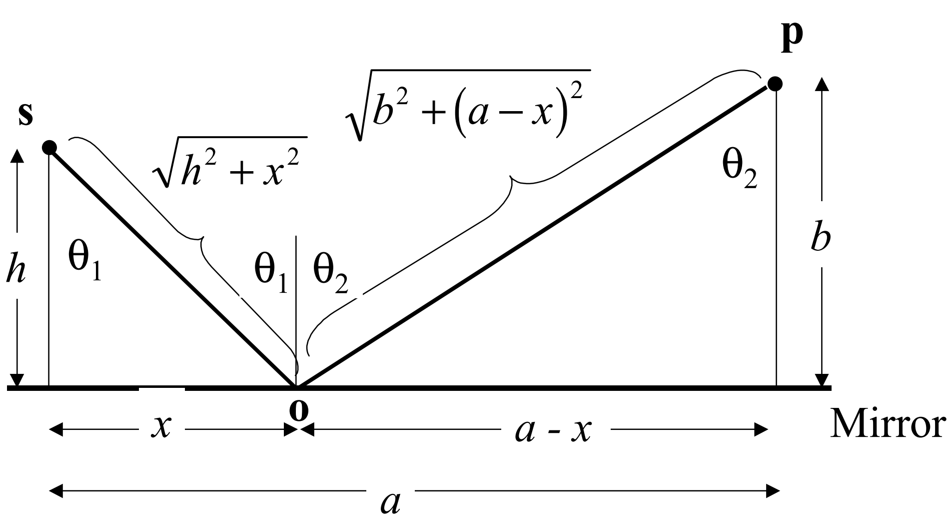
Thus the time traveled to traverse the path through the media is equal to the time required to travel a *longer path* ℓ in vacuum; the path is longer because $n_m \geq 1$. This longer path ℓ is called the *optical path length.* This means that the light requires the least time to traverse the path with the shortest optical path length:

A ray traverses the route with the shortest optical path length.

This result may be derived from Maxwell's equations.

Fermat's Principle for Reflection

Now consider the path traveled upon reflection that minimizes the simple case of optical path length:



Schematic for determining the angle of reflection using Fermat's principle

As drawn, the angle θ_1 is positive (measured from the normal) and θ_2 is negative. The ray travels in the same medium of index n both before and after reflection. The optical path length is:

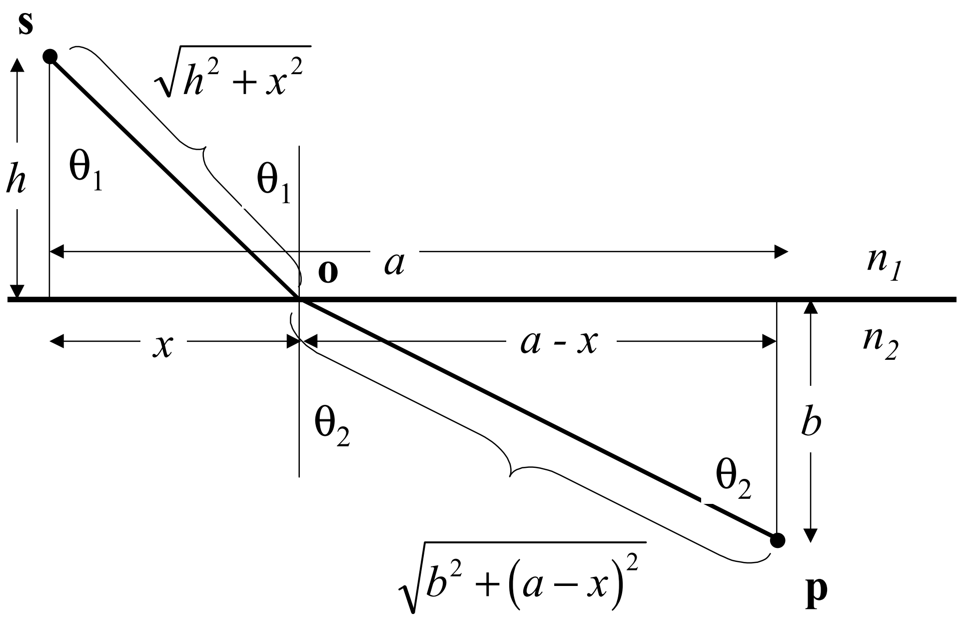
$$\begin{aligned}
\overline{\mathbf{so}} &= \sqrt{h^2+x^2} \\
\overline{\mathbf{op}} &= \sqrt{b^2+(a-x)^2} \\
\ell &= n\left(\overline{\mathbf{so}}+\overline{\mathbf{op}}\right) \\
&= n\left(\sqrt{h^2+x^2}+\sqrt{b^2+(a-x)^2}\right) \\
&\quad \textit{which is a function of } x
\end{aligned}$$

By Fermat's principle, the path length traveled is the minimum of of the optical path length ℓ, so the position of $\mathbf{o}$ may be find by setting the derivative of ℓ with respect to x to zero:

$$\begin{aligned}
\frac{d\ell}{dx} &= n\left(\frac{2x}{2\sqrt{h^2+x^2}}+\frac{-2(a-x)}{2\sqrt{b^2+(a-x)^2}}\right)=0 \\
&= \frac{x}{\sqrt{h^2+x^2}}-\frac{a-x}{\sqrt{b^2+(a-x)^2}} \\
&\Longrightarrow \frac{x}{\sqrt{h^2+x^2}}=\frac{a-x}{\sqrt{b^2+(a-x)^2}} \\
&\quad \textit{from the drawing, note that} \\
\sin[\theta_1] &= \frac{x}{\sqrt{h^2+x^2}} \\
\sin[-\theta_2] &= \frac{a-x}{\sqrt{b^2+(a-x)^2}} \\
&\Longrightarrow \sin[\theta_1]=\sin[-\theta_2] \\
&\Longrightarrow -\theta_1=\theta_2
\end{aligned}$$

In words, the magnitude of the angle of incidence = the magnitude of the angle of reflection. The negative sign is necessary because of the sign convention.

Fermat's Principle for Refraction:



Schematic for refraction using Fermat's principle.

In this drawing, both θ_1 and θ_2 are positive. The optical path length is:

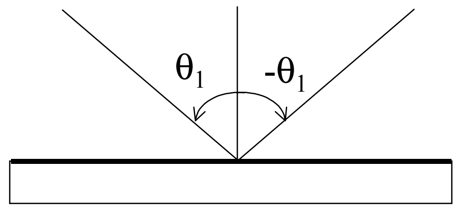
$$\begin{aligned}\ell &= n_1\,\overline{\mathbf{so}} + n_2\,\overline{\mathbf{op}}\\ &= n_1\sqrt{h^2+x^2} + n_2\sqrt{b^2+(a-x)^2}\end{aligned}$$

By Fermat's principle, the path length traveled is the minimum of ℓ, so we again set the derivative of ℓ with respect to x to zero:

$$\begin{aligned}\frac{d\ell}{dx} &= n_1\frac{2x}{2\sqrt{h^2+x^2}} + n_2\frac{-2\,(a-x)}{2\sqrt{b^2+(a-x)^2}} = 0\\ &\Longrightarrow n_1\frac{x}{\sqrt{h^2+x^2}} = n_2\frac{a-x}{\sqrt{b^2+(a-x)^2}} = 0\\ &\Longrightarrow n_1\sin[\theta_1] = n_2\sin[\theta_2]\\ \sin[\theta_1] &= \frac{x}{\sqrt{h^2+x^2}}\\ \sin[\theta_2] &= \frac{a-x}{\sqrt{b^2+(a-x)^2}}\\ &\Longrightarrow \sin[\theta_1] = \sin[\theta_2]\\ &\Longrightarrow \textit{Snell's Law for refraction}\end{aligned}$$

Note that with this sign convention, Snell's law may be applied to reflection by setting the refractive index of the second medium to be the negative of the first:

$$\begin{aligned}n_1\sin[\theta_1] &= n_2\sin[\theta_2]\\ &\Longrightarrow n_1\sin[\theta_1] = -n_1\sin[\theta_2]\\ &\Longrightarrow -\sin[\theta_1] = \sin[\theta_2]\\ &\Longrightarrow \theta_2 = -\theta_1\end{aligned}$$


θ1
-θ1