

Chapter 5

Transition from Acoustic Waves to Electromagnetic Waves

5.1 The Doppler Effect

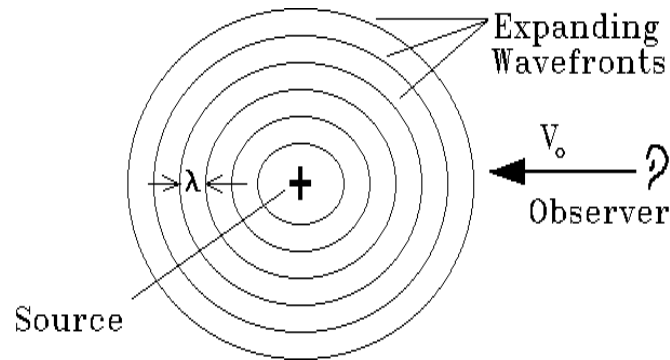
HR § 40

The change in the frequency of a sound wave due to relative motion of the source and/or receiver is very familiar – the increase in pitch of an approaching or receding locomotive airhorn is a common example. This effect was described mathematically by Christian Doppler in 1842, and is naively understood by many people. However, few realize the fundamental difference between the Doppler effect due to source motion and that due to receiver motion.

5.1.1 Acoustic Doppler Effect, Source at Rest

CASE I Motion of Receiver, Source and Medium at Rest

Consider a point source of sound in air which emits a frequency ν . The receiver moves relative to the source at velocity v_o . Since the source and medium are at rest, the sound has a wavelength $\lambda = \frac{v}{\nu}$, where v is the velocity of sound in air ($\simeq 330 \frac{m}{s}$ at *STP*). Since the source is at rest, the wavefronts expand uniformly from the source. A receiver traveling toward (away from) the source passes more (fewer) peaks of the sound wave in a given time interval than (s)he would were (s)he stationary. Therefore, the receiver hears a higher (lower) pitch.



A “snapshot” of the source, receiver, and traveling wave if the observer moves towards the source.

This is shown in a snapshot of the source, receiver, and the emerging wavefronts (*i.e.*, a wavefront is the locus of points of constant phase on a wave). The number of wave peaks heard per unit time is the observed frequency ν' , and equals the source frequency plus (minus) the number of *extra* cycles heard due to observer motion:

$$\begin{aligned}\nu' &= \nu \pm \Delta\nu = \nu \pm \frac{v_o}{\lambda} = \frac{v}{\lambda} \pm \frac{v_o}{\lambda} = \frac{v}{\lambda} \cdot \left(1 \pm \frac{v_o}{v}\right) \\ &= \nu \left(1 \pm \frac{v_o}{v}\right) = \nu'\end{aligned}$$

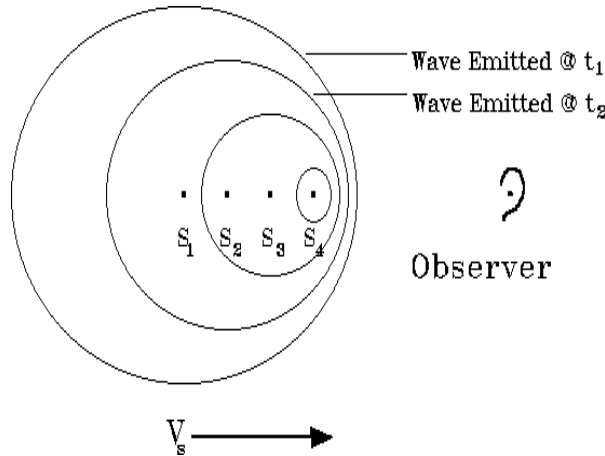
The + sign means that the receiver approaches the source.

Example: $\nu = 1000 \text{ Hz}$, $v_o = 60 \text{ mph} = 88 \text{ fps} = 26.8 \frac{\text{m}}{\text{s}}$ toward source

$$\nu' = 1000\text{Hz} \cdot \left[1 + \frac{26.8}{330}\right] \simeq 1000\text{Hz} \cdot 1.081 = 1081\text{Hz} > 1000\text{Hz}$$

5.1.2 Acoustic Doppler Effect – Source in Motion

CASE II Source Moves in Medium, Receiver Stationary



Dopper effect for sound waves if the source moves towards the observer. The circles represent wavefronts emitted by the source at different times, showing that they do not have a common center of symmetry.

Again, this is a snapshot of the wavefronts emitted by a source moving toward the receiver with velocity v_s . The wavefronts emitted at later times have less distance to travel to the observer. The distance between adjacent wavefronts *in the medium* is actually shortened on one side and lengthened on the other, *i.e.*,

$$\lambda' = \lambda \mp \Delta\lambda = \lambda \mp \frac{v_s}{\nu}, \text{ where the negative sign } \implies \text{source approaching observer.}$$

Therefore:

$$\nu' = \frac{v}{\lambda'} = \frac{v}{\lambda \mp \frac{v_s}{\nu}} = \frac{v}{\frac{\lambda\nu \mp v_s}{\nu}} = \nu \left[\frac{v}{v \mp v_s} \right] = \nu'$$

Example: $\nu = 1000 \text{ Hz}$, $v_s = 60 \text{ mph} = \frac{26.8\text{m}}{\text{S}}$ toward observer

$$\nu' = 1000\text{Hz} \left[\frac{330\frac{\text{m}}{\text{S}}}{(330\frac{\text{m}}{\text{S}} - 26.8\frac{\text{m}}{\text{S}})} \right] = 1000\text{Hz} \cdot \left[\frac{330}{303.2} \right] = 1088\text{Hz} > 1000\text{Hz}$$

In the case of the source moving in the medium, the frequency is significantly different than for the case of the observer moving (1088 Hz vs. 1081 Hz).

5.1.3 Acoustic Doppler Effect — Both Source and Receiver Moving

Case III Both Source and Receiver Moving, Medium at Rest

If both source and receiver move, the frequency is a combination of the two results:

$$\nu' = \nu \cdot \left[\frac{v \pm v_o}{v \mp v_s} \right]$$

upper signs \implies source and receiver approach each other

5.2 Doppler Effect for Light – Difference between Light and Sound

Because the Doppler effect for sound differs if the source moves rather than the observer, it is possible to determine which is moving relative to the medium. If the observer moves, the wavelength λ in

the medium is invariant and the change in pitch is due to the more-or-less frequent passages of the wavefronts by the observer. If the source moves, the wavelength of the sound in the medium changes and the sign of the change depends on the direction of source motion. If this new wavelength is λ' , then the new frequency is $\nu' = \frac{v_\phi}{\lambda'}$.

For light waves (electromagnetic radiation), the mechanism of wave propagation (and hence of the Doppler effect) is fundamentally different from propagation of sound in air. Because of this *big* difference, light propagation was not successfully described until 1864, when James Clerk Maxwell collected and interpreted the four equations which bear his name. The true nature of light was not generally accepted until post-1880. Why is light so different?

Recall that two forces are required to sustain oscillations or propagate waves – (1) inertia; (2) restoring force. Waves in common everyday experience (*e.g.*, sound in air, surface waves in water), inertia is supplied by the source (air motion from the diaphragm, physical displacement of the water surface). The restoring force is due to *a characteristic of the medium of transmission* (*e.g.*, air pressure, gravity plus surface tension).

By the early 1800's, some characteristics of light were already known, *e.g.*, the phase velocity c was known to be finite. The first recorded experiment to measure c was performed by Galileo around 1600. He stationed a man with a shuttered lantern on a distant hill with instructions to open the shutter as soon as he saw the light from Galileo's lamp. By timing the interval between unshuttering his lamp and seeing the return beam, Galileo tried to measure c via $c = \frac{2L}{t}$, where L is the distance between lanterns. His conclusion:

“If not instantaneous, light is extraordinarily rapid.”

A surprisingly good measurement of c was made by Ole R mer in 1675. The Keplerian laws of planetary motion enabled R mer to predict the times of eclipse of Jupiter's Galilean satellites. He found that the measured times did not agree with prediction – when Jupiter was closest to earth, the times of eclipse were early, and when Jupiter was distant the times were late. R mer ascribed the difference to a finite velocity of light, and computed a value of $c = 2 \cdot 10^8 \frac{m}{s}$. The largest source of error was R mer's lack of knowledge about the earth's orbital velocity. When corrected for this error, R mer's method yields a very accurate value of $3 \cdot 10^8 \frac{m}{s}$. Besides its velocity, light had been demonstrated to have the character of a wave by Newton's demonstration of dispersion by a prism and by the polarization experiments of Fresnel. These characteristics led to Fresnel's hypothesis of the “aether” – the medium of transmission for light, which is analogous to air for propagation of sound. If it exists, the aether must be present everywhere, including in vacuum. From the calculations of the Doppler effect, the frequency shift of light must depend on whether the source or the observer is moving.

The need for the aether was eliminated by Maxwell (as we shall soon see), and its existence was disproved by Michelson and Morley in 1880 when they demonstrated that the velocity of light does not vary if measured parallel to or perpendicular to the orbital motion of the earth.

Einstein used Michelson's results to derive the Special Theory of Relativity, which states:

*“The velocity of light is constant, regardless of the motion of the source or the observer.
In addition, there is no preferred frame of reference.”*

Therefore when considering light, the Doppler effect should yield identical results if the source is moving or if the observer is moving. In fact, it is impossible to define which moves; only the *relative* motion is meaningful.

Einstein's result is:

$$\nu' = \nu \left(\frac{1 - \frac{v}{c}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \right) = \left[v \left(1 - \frac{v}{c} \right) \right] \left(1 - \left(\frac{v}{c} \right)^2 \right)^{-\frac{1}{2}}$$

which may be approximated by applying the power series:

$$(1 + u)^n = \frac{1}{0!} + \frac{n}{1!}u + \frac{n(n-1)}{2!}u^2 + \frac{n(n-1)(n-2)}{3!}u^3 + \dots + \left(\frac{n!}{(n-r)!} \right) \frac{1}{r!}u^r + \dots$$

to obtain:

$$\begin{aligned}
 \nu' &= \left[v \left(1 - \frac{v}{c} \right) \right] \left(1 - \frac{1}{2} \left(\frac{v}{c} \right)^2 + \left(-\frac{1}{2} \right) \left(-\frac{3}{2} \right) \frac{1}{2!} \left(\frac{v}{c} \right)^2 + \dots \right) \\
 &= \left[v \left(1 - \frac{v}{c} \right) \right] \left(1 - \frac{1}{2} \left(\frac{v}{c} \right)^2 + \frac{3}{8} \left(\frac{v}{c} \right)^4 + \dots \right) \\
 &= \nu \left(\left[1 - \left(\frac{v}{c} \right) \right] + \left[\frac{1}{2} \left(\frac{v}{c} \right)^2 - \frac{1}{2} \left(\frac{v}{c} \right)^3 \right] + \left[\frac{3}{8} \left(\frac{v}{c} \right)^4 - \frac{3}{8} \left(\frac{v}{c} \right)^5 \right] + \dots \right) \\
 \nu' &\simeq \nu \left(1 - \frac{v}{c} \right) = \nu - \nu \frac{v}{c} \text{ for } v \ll c.
 \end{aligned}$$

Thus the Doppler shift decreases the frequency (and increases the wavelength) if v is positive (distance between source and observer increases) by an amount that is proportional to v . This is the famous “red shift” in astronomy.

