1051-232-20033 Imaging Systems Laboratory II
Week of 5/3/2004, Writeup Due 5/18/2004 (T)

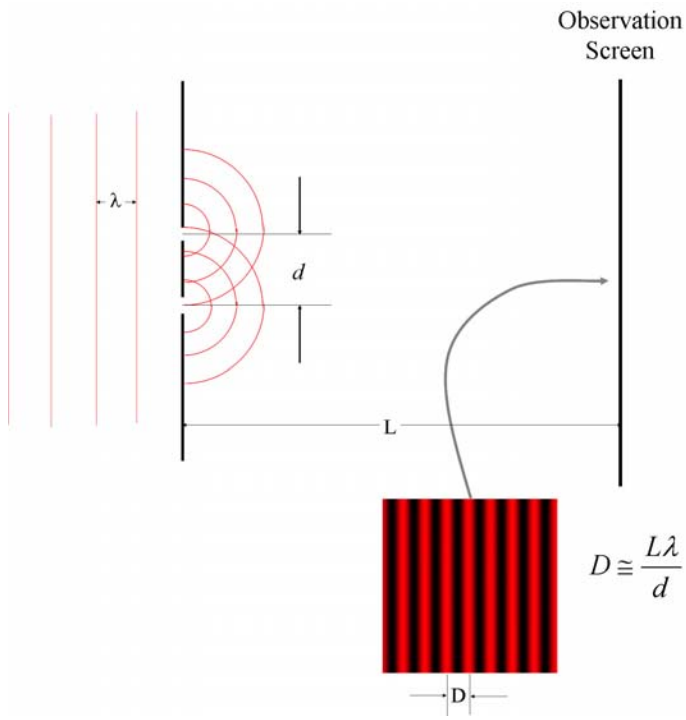
1 Laboratory 8: Michelson Interferometer

1.1 Theory:

References: Optics, E. Hecht, sec 9.4

1.1.1 Division-of-Wavefront Interferometry

In the last lab, you saw that *coherent* (single-wavelength) light from point sources two different locations in an optical beam could be combined after having traveled along two different paths. The recombined light exhibited *sinusoidal fringes* whose spatial frequency depended on the difference in angle of the light beams when recombined. The regular variation in relative phase of the light beams resulted in *constructive* interference (when the relative phase difference is $2\pi n$, where n is an integer) and *destructive* interference where the relative phase difference is $2\pi n + \pi$. The centers of the "bright" fringes occur where the optical paths differ by an integer multiple of λ_0.



"Division-of-Wavefront" Interferometry via Young's Two-Slit Experiment. The period of the intensity fringes at the observation plane is $D \simeq \frac{L\lambda}{d}$.

1.1.2 Division-of-Amplitude Interferometry

This laboratory will introduce a second class of interferometer where the amplitude of the light is separated into two (or more) parts *at the same point in space* via partial reflection by a *beamsplitter*. The beamsplitter is the light-dividing element in several types of interferometers. This lab will demostrate the action of the Michelson interferometer for both coherent light from a laser and for white light (with some luck!).

Coherence of a Light Source If two points within the same beam of light exhibit a consistent and measurable phase difference, then the light at the two points is said to be *coherent*, and thus the light at these two locations may be combined to create interference. The ease with which fringes can be created and viewed is determined by the *coherence* of the source. The *coherence length* is the difference in distance travelled by two beams of light such that they generate detectable interference when recombined. The coherence *time* is the time period that elapses between the passage of these two points that can "just" interfere:

$$\ell_{coherence} = c \cdot t_{coherence}$$

The coherence time (obviously) has dimensions of time (units of seconds). Light emitted by a laser spans a very narrow range of wavelengths, and may be approximated as emitting a single wavelength λ_0. The temporal "bandwidth" of a light source is the range of emitted temporal frequencies:

$$\begin{aligned}
\Delta\nu &= \nu_{\max} - \nu_{\min} \\
&= \frac{c}{\lambda_{\min}} - \frac{c}{\lambda_{\max}} = c \cdot \left(\frac{1}{\lambda_{\min}} - \frac{1}{\lambda_{\max}}\right) \\
&= c \cdot \left(\frac{\lambda_{\max} - \lambda_{\min}}{\lambda_{\max} \cdot \lambda_{\min}}\right) \\
&= \frac{c \cdot \Delta\lambda}{\lambda_{\max} \cdot \lambda_{\min}} \left[\frac{\text{cycles}}{\text{second}} = \text{Hz}\right]
\end{aligned}$$

For a laser, $\lambda_{\max} \simeq \lambda_{\min} \simeq \lambda_0$ and $\Delta\lambda \simeq 0$, so the temporal bandwidth is very small:

$$\Delta\nu \simeq \frac{c \cdot 0}{\lambda_0^2} \to 0$$

The reciprocal of the temporal bandwidth has dimensions of time and is the *coherence time*.

$$\begin{aligned}
t_{coherence} &= \frac{1}{\Delta\nu}\,[\text{s}] \to \infty \text{ for laser} \\
\ell_{coherence} &= \frac{c}{\Delta\nu}\,[\text{m}] = \frac{\lambda_{\max} \cdot \lambda_{\min}}{\Delta\lambda} \to \infty \text{ for laser}
\end{aligned}$$

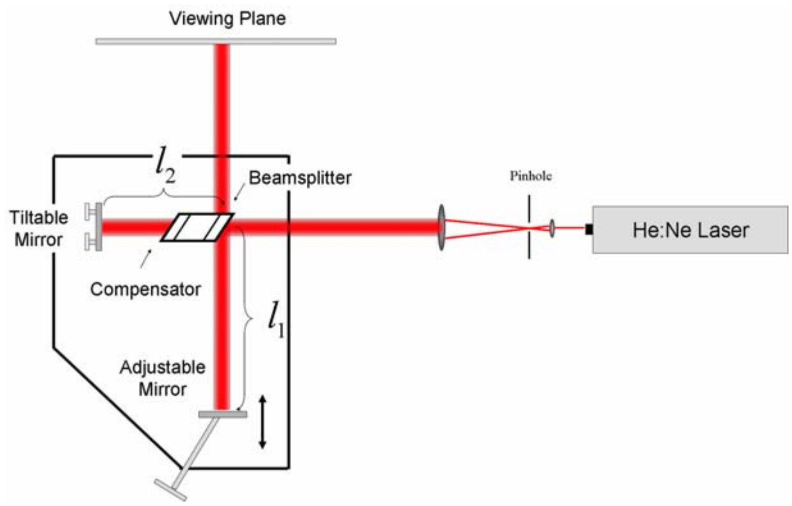
These results demonstrate that light from a laser can be delayed by a very long time and still produce interference when recombined with "undelayed" light. Equivalently, laser light may be sent down a very long path and then recombined with light from the source and still produce interference (as in a Michelson interferometer).

Question: Compute the coherence time and coherence length for white light, which may be modeled as containing "equal amounts" of light with all frequencies from the blue ($\simeq 400\,\text{nm}$) to the red ($\lambda \simeq 700\,\text{nm}$). You will find these to be very small, and thus it is MUCH easier to create fringes with a laser than with white light. The coherence length of the source may be increased by filtering out many of the wavelengths, which also reduces much of the available intensity!

1.1.3 Michelson Interferometer

The Michelson interferometer was used by Michelson (surprise!) when he showed that the velocity of light does not depend on the direction travelled. The Michelson interferometer apparatus is shown in the figure below. Unfortunately, we only have one apparatus, so that teams will have to "rotate" through the experiment.

Collimated Laser Light The light is split into two beams by the beamsplitter, which is half-silvered on the inside so that light is reflected after being transmitted through the glass. The two beams emerging from the beam splitter travel perpendicular paths to mirrors where they are directed back to be recombined at the beamsplitter surface. One mirror can be tilted by an adjustable mirror and the length of the other path may be changed by a translatable mirror. A *compensator* (a plain piece of glass of the same thickness as the beamsplitter) is placed in the beam that was transmitted by beamsplitter. The light transits the compensator twice, and thus the path length traveled in glass by light in this arm is identical to that traveled in glass in the other arm. This equalization of the path length traveled simplifies (somewhat) the use of the interferometer with a broad-band white-light source. The compensator is not necessary when a laser is used, because the coherence length is much longer than the path-length difference.



Michelson interferometer using He:Ne laser for illumination ($\lambda_0 = 632.8\,\text{nm}$).

If the translatable mirror is displaced so that the two beams travel different distances from the beamsplitter to the mirror (say ℓ_1 and ℓ_2), then the total optical path of each beam is $2 \cdot \ell_1$ and $2 \cdot \ell_2$. The total *optical path difference* is:

$$OPD = 2 \cdot \ell_1 - 2 \cdot \ell_2 = 2 \cdot \Delta\ell \quad [\,\text{m}]$$

Note that the path length is changed by TWICE the translation distance of the mirror. The OPD may be scaled to be measured in number of wavelengths of difference simply by dividing by the laser wavelength:

$$OPD = \frac{2 \cdot \Delta\ell}{\lambda_0} \quad [\text{wavelengths}]$$

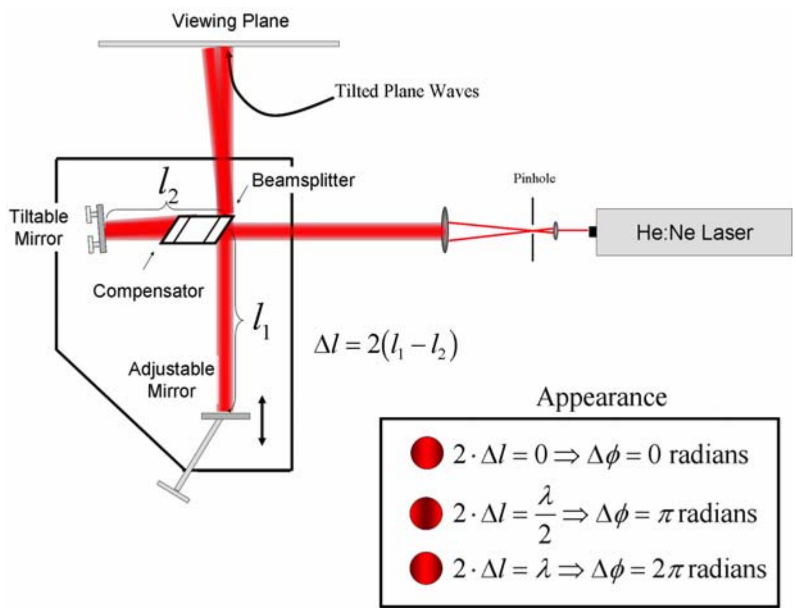
We know that each wavelength corresponds to 2π radians of phase, so the *optical phase difference* is the OPD multiplied by 2π radians per wavelength:

$$\begin{aligned} O\Phi D &= 2\pi \left[\frac{\text{radians}}{\text{wavelength}}\right] \cdot \frac{2 \cdot \Delta\ell}{\lambda_0} \quad [\text{wavelengths}] \\ &= 2\pi \cdot \frac{2 \cdot \Delta\ell}{\lambda_0} \quad [\text{radians}] \end{aligned}$$

If the optical phase difference is an integer multiple of 2π (equivalent to saying that the optical path difference is an integer multiple of λ_0), then the light will combine "in phase" and the interference is *constructive*; if the optical phase difference is an odd-integer multiple of π, then the light combines "out of phase" to destructively interfere.

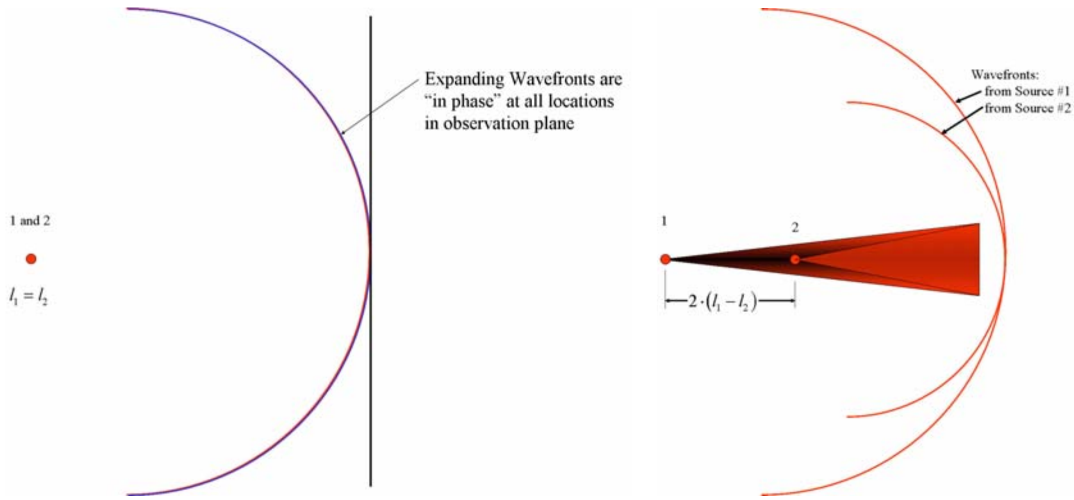
The Michelson interferometer pictured above uses a collimated laser source (more properly called a *Twyman-Green* interferometer), the two beams are positioned so that all points of light are recombined with their exact duplicate in the other path except for (possibly) a time delay if the optical paths are different). If the optical phase difference of the two beams is $2\pi n$ radians, where n is an integer, then the light at all points recombines in phase and the field should be uniformly "bright". If the optical phase difference is $(2n+1)\,\pi$ radians, then light at all points recombines "out of phase" and the field should be uniformly "dark".

If the collimated light in one path is "tilted" relative to the other before recombining, then the optical phase difference of the recombined beams varies linearly across the field in exactly the same fashion as the Young's two-slit experiment described in the handout for the last laboratory. In other words, the two beams travel with different wavevectors $\underline{\mathbf{k}}_1$ and $\underline{\mathbf{k}}_2$ that produce the linear variation in optical phase difference. With the Michelson, we have the additional "degree of freedom" that allows us to change the optical path difference of the light "at the center". As shown in the figure, if the optical path difference at the center of the observation screen is 0, then the two beams combine in phase at that point. If one path is lengthened so that the optical phase difference at the center is π radians, then the light combines "out of phase" to produce a dark fringe at the center.



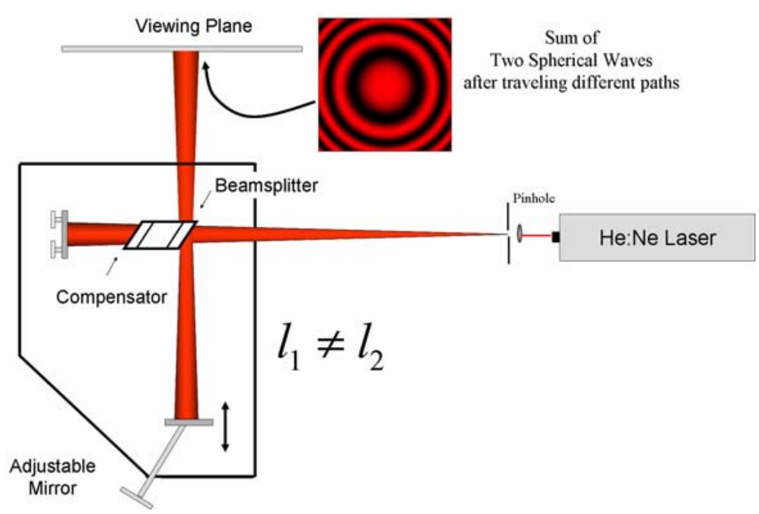
Michelson interferometer with collimated light so that one beam is tilted relative to the other. The optical phase difference varies linearly across the field, producing linear fringes just like Young's two-slit interference.

Interference of Expanding Spherical Waves The strict definition of a Michelson interferometer assumes that the light is an expanding spherical wave from the source, which may be modeled by deleting the collimating lens. If optical path difference is zero (same path length in both arms of the interferometer), then the light recombines in phase at all points to produce a uniformly bright field, as shown on the left. However, in general the optical path lengths differ so that the light from the two sources combine at the center with a different phase difference, as shown on the right.



$OPD = 0$ $\qquad\qquad$ $OPD \neq 0$

Thus the optical path difference varies more quickly at the edges of the observation plane than at the center, so the fringe period varies with position. In the figure, the light waves recombine “in phase” at the centger of the observation plane, but the period of the fringes decreases with increasing distance from the center.



1.2 Procedure:

1. (Do ahead of the lab, if possible) Consider the use of the Michelson interferometer with a point source of light (expanding spherical waves) where one mirror is “tilted” and the path lengths are equal. Redraw the optical configuration of the Michelson interferometer to show the separation of the “effective” point sources due to the optical path difference in the two arms. The spherical wavefronts generated by the images of the point source in the two arms interfere to produce the fringes. Explain the shape of the fringes seen at the output.

2. (Do ahead of the lab, if possible) Repeat for the case of a point source where the two mirrors are “aligned” so that the beams are not tilted. Again, the spherical wavefronts generated by the images of the point source in the two arms interfere to produce the fringes. Explain the shape of the fringes seen at the output.

3. In the first part of the lab, use a lens to expand the laser beam into a spherical wave. The mirrors generate images of the source of the spherical wave, and the resulting spherical wave-

fronts superpose. The squared magnitude of the spatial modulation of the superpostion is the visible fringe pattern. The drawings below give an idea of the form of the fringes that will be visible for various configurations of the two mirrors when used with spherical waves.

4. Once you have obtained circular fringes that are approximately centered, move the translatable mirror to increase or decrease the optical path difference. (be sure that you know which!). Note the direction that the fringes move; in other words, do they appear from or disappear into the center? Explain.

5. Note that fringes also are produced at the input end of the interferometer. How do they differ from those at the output end?

6. Place polaroid filters in each path and note the effect on the interference pattern as the relative polarizations of the beams are changed. Also try the same experiment with $\lambda/4$ plates, $\lambda/2$ plates, and the circular polarizer.

7. Place a piece of plastic wrap in one arm of the interferometer. Describe and explain its effect.

8. Place a spherical lens (best if the focal length f is very long) in the beam at the input end and note the effect on the fringes. Repeat with the lens in one arm.

9. Put a source of heat in or under one of the arms of the interferometer; your hand will work (if you are warmblooded!), but a more intense source such as a soldering iron works better. Note the effect.

10. The wavefronts in the second part of the lab should be approximately planar; use a second lens to generate *collimated* light. See if you can generate a pattern that is all white or all black, in other words, the pattern is a single bright or dark fringe. When you have generated the dark fringe, try the experiment with the heat source in one arm again.

1.3 Questions:

1. (Already mentioned in the text) Compute the coherence time and coherence length for white light, which may be modeled as containing "equal amounts" of light with all frequencies from the blue ($\simeq 400\,\text{nm}$) to the red ($\lambda \simeq 700\,\text{nm}$).

2. (Already mentioned in the text) Redraw the optical configuration of the Michelson to show the separation of the effective point sources due to the path-length difference in the two arms of the interferometer.

3. Explain the direction of motion of the circular fringes when the path length is changed, i.e., what directions do the circular fringes move if the OPL is increased? What if OPL is decreased?.

4. When the Michelson is used with collimated light, explain how a single dark fringe can be obtained. Where did the light intensity go?

5. Explain what happens when a piece of glass (or other material) is placed in one arm.