

# 1 Thin Lenses and Thin Lens Combinations

## 1.1 Abstract:

This laboratory considers several concepts in geometrical optics and thin lenses: the assumption of rectilinear propagation (light as rays) that defines geometrical optics; the mapping from object space to image space by a thin lens (including real and virtual images, longitudinal and transverse magnifications, and the differential radiometry of the mapping); and the two types of cardinal points of lenses (focal and principal points). In particular, we will verify the thin lens formula and the formula for the focal length of two lenses combined, examine image size and fidelity, and construct both Galilean and Keplerian telescopes. As a side benefit, the lab will also demonstrate (though not quantify) the variation in image quality for on-axis and off-axis objects due to aberrations (primarily chromatic aberration, spherical aberration and coma), and demonstrate the effect of stops and the lens shape factor on image quality.

## 1.2 Theory:

Thin lenses are used in many optical systems, and understanding how they work involves many basic concepts in geometrical optics. These include:

1. the assumption of “rectilinear propagation ”(light as “rays”),
2. the mapping from object space to image space
3. the variation in quality of images of “on-axis ”and “off-axis ”objects due to aberrations present in the lens (primarily chromatic aberration, spherical aberration and coma),
4. the effect of stops and the lens “shape factor ”on image quality.

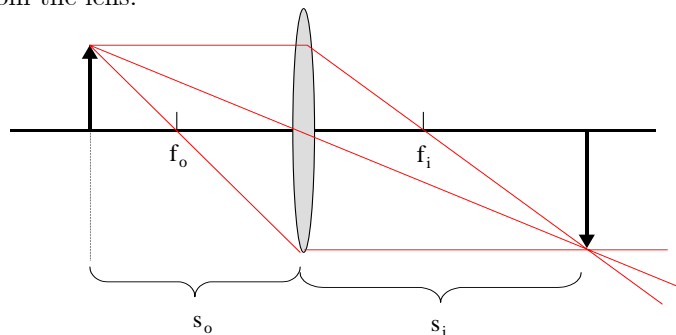
Consider the imaging equation for a single thin lens:

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \quad (1)$$

where  $s_o$  and  $s_i$  are object and image distances from the lens, respectively, and  $f$  is the focal length of the lens. By simple rearrangement, we can express  $s_i$  as a function of  $s_o$  and  $f$ :

$$s_i = \frac{s_o f}{s_o - f} \quad (2)$$

Note that this relationship is extremely nonlinear: the image distance is definitely NOT proportional to the object distance. This relationship will be demonstrated by imaging the white light source at various distances from the lens.



*Ray diagram for a single thin lens: an input ray parallel to the optical axis passes through the rear focal point, an input ray through the center of the lens emerges at the same angle, and an input ray that passes through the front focal point emerges parallel to the optical axis.*

The transverse magnification of the image (compared to the object) is the ratio of the image distance to the object distance

$$M_T = -\frac{s_i}{s_o} \quad (3)$$

(one can easily derive this result by analyzing similar triangles in Figure 1), where the negative sign indicates that the image is inverted if both  $s_o$  and  $s_i$  are positive.

For combinations of two thin lenses in contact, we have seen that the focal length of the combination is given by an expression involving the individual focal lengths:

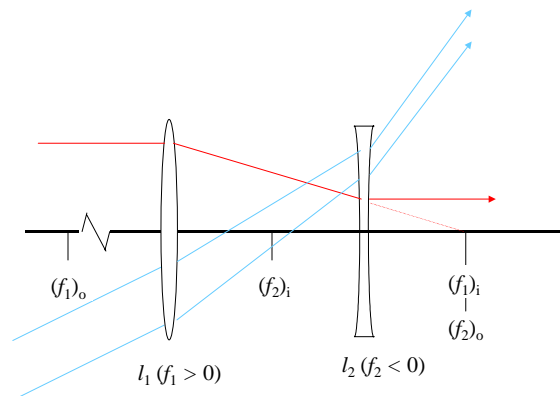
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \quad (4)$$

Combinations of two thin lenses can be used to make an extremely important optical device: the refracting telescope. Although it is not totally clear who invented the telescope, records in The Hague (in Holland) demonstrate that Hans Lippershey (1587-1619) applied for a patent on the device in 1608. Soon after, Galileo built his first telescope using one positive lens with convex surfaces and one negative lens with concave surfaces. Kepler also used a telescope, but favored an arrangement with two convex lenses. In both cases, the lens where the light enters is called the “objective lens”, while and the lens where the user places their eye is the “eye lens.” Both the Galilean and Keplerian designs are shown in Figure 2. They differ in that the position of the eye lens as well as the type of lens. The Keplerian telescope forms a real intermediate image, while the Galilean does not. Note that both designs are “afocal”; the systems do not form accessible “images”. Rays that enter at an angle  $\theta$  measured relative to the optical axis emerge at the same angle, so parallel entering rays emerge in parallel. A third lens (usually your eye lens or a camera) forms the real image. The telescope is formed when the two lenses are separated by a distance equal to the sum of their focal lengths:  $d = f_1 + f_2$ . The focal length of the resulting system is obtained by substitution into the equation:

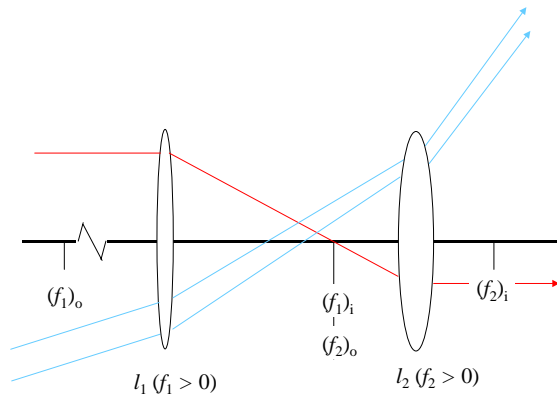
$$\begin{aligned} \frac{1}{f} &= \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \\ &= \frac{1}{f_1} + \frac{1}{f_2} - \frac{f_1 + f_2}{f_1 f_2} = 0 \text{ mm}^{-1} \implies f = \infty \text{ mm, for a telescope} \end{aligned} \quad (5)$$

:The “power” of the system (reciprocal of the system focal length) is:

$$\frac{1}{f} = \varphi = \varphi_1 + \varphi_2 - \varphi_1 \varphi_2 d = 0 \text{ mm}^{-1}, \text{ for a telescope} \quad (6)$$



*Optical layout of Galilean telescope: the “image space” (rear) focal point of the positive objective lens coincides with the “object space” focal of the negative eye lens. Light rays that are incident parallel to the axis emerge parallel to the axis. Parallel rays that enter the object at an angle to the optical axis emerge parallel but at a “steeper” angle to the axis.*



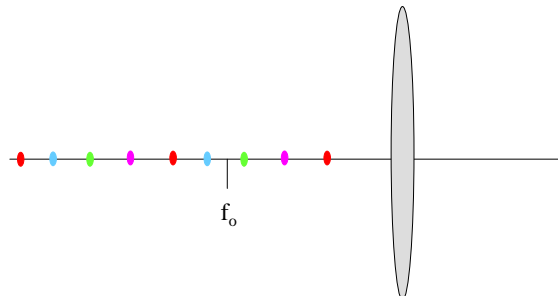
Optical layout of Keplerian telescope: the “image space” (rear) focal point of the positive objective lens coincides with the “object space” focal of the positive eye lens. Light rays that are incident parallel to the axis cross the axis at the focal point of the objective lens and emerge parallel to the axis and on the “other” side. Parallel rays into the object at an angle relative to the optical axis emerge parallel and at a “steeper” angle.

### 1.3 Procedure:

#### 1.3.1 Demonstration:

The first part of this lab is a demonstration of images generated by a large plano-convex singlet lens, which illustrates the geometrical mapping of different points in *object space* to different locations in *image space* by the action of the lens. The particular lens suffers from significant amounts of both chromatic and monochromatic aberrations, and thus generates images that significantly differ from the point-to-point character often assumed in geometrical optics.

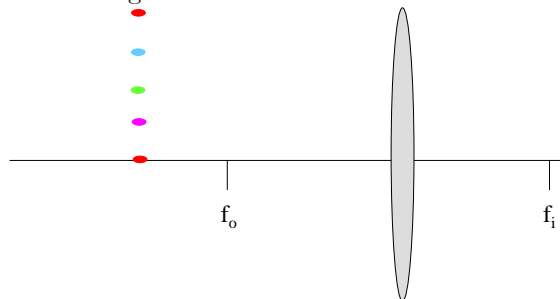
1. The lens will be used first to generate images of a string of colored Christmas lights stretched along the optical axis. Use a white card for reflective viewing or a diffuser (ground glass or tissue paper) for transmissive viewing of the *real* images. The different colored lights should be helpful in locating the images corresponding to specific locations in object space. Measure the distances from the lights and from the images to the same point located as near as possible to the center of the lens. Use the ensemble of measurements of the real object/image distances to estimate the focal length of the lens and the error of this measurement. You may also try to estimate the longitudinal magnification and the brightness of the images for different locations along the optical axis.



Christmas lights along the optical axis

2. If the string of Christmas lights is set up *perpendicular* to the optical axis, the locations of the corresponding images are determined by the transverse magnification of the imaging system. Measure the distances along both axes (along and transverse to the optical axis) for both the

object and corresponding image. If time allows, you should repeat this step for several lines of lights at different distances along the optical axis. From these measurements, you can construct a plot of the *transverse magnification* at different points in the image space. Also, note the visual quality of real images off axis; that is, whether the images of off-axis lights differ significantly from those of on-axis lights. Sketch the differences.

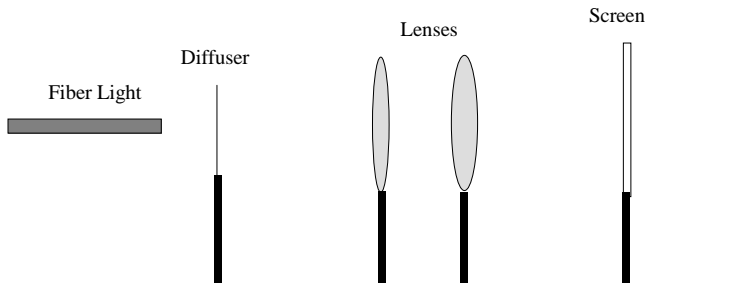


*Christmas lights across optical axis*

3. The lens may be “stopped down” with an opaque shield to reduce its effective diameter. Compare the “quality” of the images obtained from the “full-aperture” lens and from a “stopped-down” lens.
4. Cover the lens with the array of regularly spaced holes and look at the array of images at different locations along the optical axis (this is a crude version of the common *Hartmann test* for optical systems). The aberrations of the lens should be apparent from the appearance of the images of the colored lights (particularly for images of off-axis objects). You might also try this with a laser source that has been expanded with a spatial filter.
5. By using the white-light source, examine the effects of chromatic aberration in both the longitudinal and transverse directions.

### 1.3.2 Experiment:

Obtain posts, post holders, and lens holders to mount three lenses on your optical rail. Save one of the mounts for holding a white piece of paper as an observation screen that may be placed at various positions. The Newport lens kits include 2"-diameter lenses of several focal lengths. BE CAREFUL WHEN HANDLING THE LENSES – AVOID DROPPING OR FINGERPRINTING THEM. You’ll also need a lens for the CCD camera for the telescope exercises. Although we will change the configuration from step to step in the lab, a basic configuration is shown.



*Schematic of optical setup.*

1. For one of the “unknown” lenses provided, measure seven or eight pairs of object and image distances and the size of the real image. Evaluate the transverse magnifications at each object distance from your data. Use the ensemble of measurements to estimate the focal length of

the lens and the error of this measurement. That is, calculate  $f$  for each pair of distances. Average the measurements to determine your final result and estimate the standard deviation  $\sigma$  of the measurements. Include as your uncertainty in the focal length  $\frac{\sigma}{\sqrt{N}}$ , where  $N$  is the number of distance pairs you have measured. Repeat the procedure for a second “unknown lens.”

2. Next, take two positive lenses from your kit and put them in contact as closely as possible on the optical rail. Two planar convex lenses with their flat sides in contact will probably work best. Use the same method as in Step 1, measure the focal length and transverse magnification of the combination. The focal lengths of the individual lenses are labeled in the kits, so you can calculate the expected value of  $f_{eff}$ . Within your uncertainty, is the formula satisfied?
3. For one of the “very convex” lenses (short focal length, large power) in your kit, determine image distances for two different object distances for (a) white light, (b) blue light, and (c) red light. Use filters in front of the light source to change the color. What is the difference in focal length (mm) for red and blue light?
4. Place an aperture stop just in front of the lens. How does this affect the difference in distance between the blue and red focus points? Try the same experiment with a thinner lens. How does the change affect the difference in distance?
5. Use the “very convex” lens used in the previous step to study the image for an off-axis object in a qualitative sense. Sketch the effect on the image as the object is moved away from the optical axis.
6. Using Figure 2 as a guide, set up a Galilean telescope. Use a lens with a “long ” positive focal length for the objective and an “short ” negative focal length for the eye lens. Try to arrange things so your telescope points out the door of the lab. Take a look through it and determine if the image of a distant object is magnified. Since you won’t really be looking at an object at infinity, you will have to adjust the position of the eye lens slightly to compensate. Set up the CCD camera behind your telescope. The lens attached to the camera will act as the third system lens to form the image. Grab an image of something recognizable outside the door.(Try taping a sheet of paper with some writing on it to the wall opposite the door.) Now remove telescope and grab another image of the same target. What magnification did you obtain with your telescope/camera setup? Note the orientation of the image in both cases. Save your images and include them with your report.
7. Repeat the last step for a Keplerian telescope using an objective with focal length at least twice that of the eye lens.

## 1.4 Analysis:

In your lab write-up, be sure to include the following:

1. From your data in Step 1 of #2, plot  $s_i$  as a function of  $s_o$ . Estimate uncertainties in both  $x$ - and  $y$ -coordinates and plot these as error bars. Overplot the curve that you get using the average value of the focal length you measured.
2. From your data in Step 2 of #2, again plot  $s_i$  as a function of  $s_o$  with error bars. This time, overplot the curve you get using the value of  $f_{eff}$  calculated from the equation. Do your data match the predicted curve?
3. Note that if we plug the right side of Equation 2 into Equation 3, we obtain

$$M_T = -\frac{f}{s_o - f} \quad (7)$$

For the data obtained in steps 1 and 2, plot the measured image magnification measured as a function  $s_o$  and then overplot the curve expected from the above equation using the focal lengths you have measured. Do the data and the expected curve agree?

## 1.5 Questions:

1. Plot the graph of  $z_2$  vs.  $z_1$  for  $z_1$  in the range  $-10f \leq z_1 \leq +10f$ .
  - (a) Confirm the (very useful) series identity:

$$\begin{aligned} \frac{1}{1-u} &= \sum_{n=0}^{\infty} u^n = u^0 + u^1 + u^2 + \dots + u^N + \dots \\ &= 1 + u + u^2 + u^3 + \dots \text{ for } |u| < 1 \end{aligned}$$

by computing a partial sum for a few values of  $u$  (e.g.,  $u = \frac{1}{10}, \frac{1}{2}$ , etc.).

- (b) Use this result to find a series expression for  $z_2$  in eq.(1.2) in terms of  $f$  and  $z_1$  to illustrate the nonlinear relationship, assuming that  $|z_1| > |f|$ .
  - (c) In the (usual) limit where  $z_1 \gg f$  (meaning that the object distance is much larger than the focal length), the infinite series can be approximated by the sum of a few terms, where “few” is 2 – 4. Demonstrate the validity of the approximation. (This exercise, though not specifically optical, is intended to relate the subject to some of the mathematics that is essential for many imaging problems.)
2. Plot the image space along the along the optical axis, *i.e.* locate the set of image points that are generated by a set of object points that are equally spaced points along the optical axis.
  3. Consider the case where Christmas lights are extended along the optical axis for the big lens. Sketch the situation where a green lamp is closer to the lens and “obscuring” of a red lamp as seen by the lens. Do you expect that an image of the red lamp will be seen in image space (in other words, is the image of the red lamp “blocked”)? Explain.