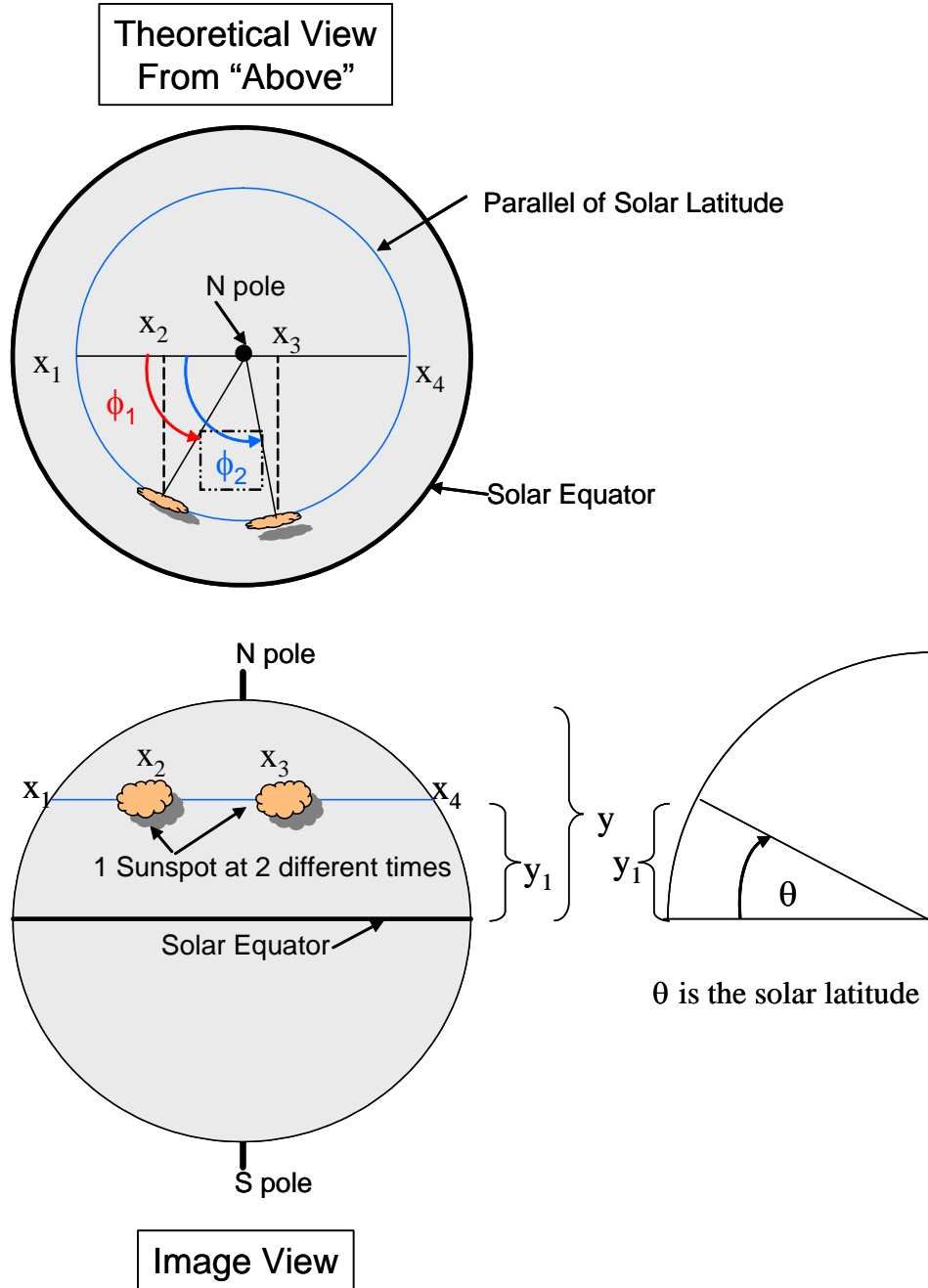


1 Hints on Sunspot Measurements – Lab 3

The calculations in the sunspot lab are a little trickier than they might first appear. This addendum to the lab writeup is intended to help guide you through the calculations. The picture of the sun is shown below:



The images of the same sunspot at two different times are shown. The angles are defined as: θ is the solar latitude of the feature; ϕ_1 is the angle in solar longitude of the first spot image measured from the left "edge" of the sun; ϕ_2 is the angle in solar longitude of the second spot image.

Sunspots travel along a line of solar latitude, i.e., they are always located at the same “height” measured from the solar equator (or equivalently, from the solar pole). Along that line of solar latitude, we can identify 4 locations *along that parallel of solar latitude* that are distinguished by their x -coordinates:

x_1	<i>coordinate of left “edge”</i>
x_2	<i>coordinate of sunspot in first image</i>
x_3	<i>coordinate of sunspot in second image</i>
x_4	<i>coordinate of right “edge”</i>

We now need to figure out the angle ϕ_1 from the left edge to the first image of the spot (shown in red in the figure) and the angle ϕ_2 from the left edge to the second image of the spot. The difference between these two angles:

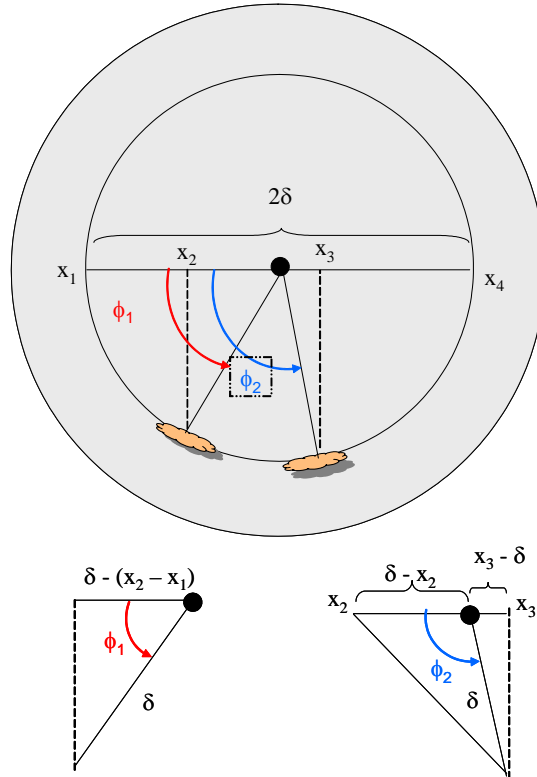
$$\Delta\phi \equiv \phi_2 - \phi_1$$

is the angle of rotation of the sun between the times of the two images.

First, find the “semidiameter” of the sun in *pixels* (which we could also call the “radius”) at that latitude; I’ll call it δ . From the figure, this is obviously:

$$2\delta \equiv x_4 - x_1 \implies \delta = \frac{x_4 - x_1}{2}$$

From the second figure, we can see that the angle ϕ_1 satisfies the trigonometric relation:



The trigonometric relations for the two angles of the sunspots measured from the point on the edge of the sun where they would appear. The two triangles shown at the bottom are used to derive the formulas for the angles.

$$\begin{aligned}\cos[\phi_1] &= \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{\delta - (x_2 - x_1)}{\delta} = \frac{\frac{x_4 - x_1}{2} - (x_2 - x_1)}{\frac{x_4 - x_1}{2}} \\ &\Rightarrow \boxed{\phi_1 = \cos^{-1} \left[\frac{\delta - (x_2 - x_1)}{\delta} \right]}\end{aligned}$$

where δ is the “semidiameter” of the sun at that latitude (in pixels) and x_1 and x_2 are the coordinates of the left edge of the sun and of the sunspot measured at that same latitude.

The second angle ϕ_2 (of the sunspot measured from the left edge of the sun at that latitude) is more complicated to evaluate, but it can be seen from the figure to satisfy the trigonometric relation:

$$\begin{aligned}\cos[\pi - \phi_2] &= -\cos[\phi_2] = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{x_3 - \delta}{\delta} \\ &\Rightarrow \boxed{\phi_2 = \cos^{-1} \left[\frac{\delta - x_3}{\delta} \right]}\end{aligned}$$

The difference in these two angles is the angle traveled by the spot between the two images:

$$\begin{aligned}\Delta\phi &\equiv \phi_2 - \phi_1 \\ &= -\cos^{-1} \left[\frac{\delta - x_3}{\delta} \right] - \cos^{-1} \left[\frac{\delta - (x_2 - x_1)}{\delta} \right]\end{aligned}$$

The rotation period of the sunspot at this latitude is the time it would take for the spot to go around the full 360° of longitude at that solar latitude. You can find the times when the images were taken either from the times listed in the file name or by using the capability in DS9 to read the FITS header. Once you find those times, say t_1 and t_2 , you can calculate the time interval Δt :

$$\Delta t = t_2 - t_1$$

The “rate of change” of the longitudinal angle (measured in degrees per day) is:

$$\frac{\Delta\phi}{\Delta t} = \frac{\left(\cos^{-1} \left[\frac{x_3 - x_2}{\delta} \right] - \cos^{-1} \left[\frac{\delta - (x_2 - x_1)}{\delta} \right] \right) [\text{degrees}]}{(t_2 - t_1) [\text{days}]}$$

and the rotation period of the sun spot is:

$$\begin{aligned}T [\text{days}] &= \frac{360^\circ}{\left(\frac{\Delta\phi}{\Delta t} \right)} \\ &= \frac{360^\circ}{\left(\cos^{-1} \left[\frac{x_3 - x_2}{\delta} \right] - \cos^{-1} \left[\frac{\delta - (x_2 - x_1)}{\delta} \right] \right) [\text{degrees}]} \cdot (t_2 - t_1) [\text{days}]\end{aligned}$$

Since the sun is a gaseous sphere, rather than a solid, its rotation rate depends to some degree (pun intended) on the latitude; it rotates the fastest at the solar equator and somewhat more slowly at “higher” latitudes (closer to the pole). The rotation period at the equator is approximately 27.3 earth days and about 28.3 days at latitude 30° .

2 Example:

Consider an example (not meant to simulate real data from our images!)

Say that the following coordinates are measured along a line of latitude:

$$\begin{aligned}x_1 &= 4 \text{ pixels} \\x_2 &= 12 \text{ pixels} \\x_3 &= 19 \text{ pixels} \\x_4 &= 96 \text{ pixels}\end{aligned}$$

Thus the “semidiameter” of the sun at that latitude is:

$$\delta = \frac{x_4 - x_1}{2} = \frac{96 - 4}{2} = 46 \text{ pixels}$$

The first angle of longitude is:

$$\begin{aligned}\phi_1 &= \cos^{-1} \left[\frac{\delta - (x_2 - x_1)}{\delta} \right] \\&= \cos^{-1} \left[\frac{46 - (12 - 4)}{46} \right] = \cos^{-1} \left[\frac{19}{23} \right] \cong 0.60 \text{ radians} \\ \phi_1 &\cong 34.3^\circ\end{aligned}$$

The second angle is:

$$\begin{aligned}\phi_2 &= \cos^{-1} \left[\frac{\delta - x_3}{\delta} \right] = \cos^{-1} \left[\frac{46 - 19}{46} \right] \cong 0.94 \text{ radians} \\ &\cong 54.1^\circ\end{aligned}$$

The difference in angles is:

$$\Delta\phi \equiv \phi_2 - \phi_1 \cong 54.1^\circ - 34.3^\circ = 19.8^\circ$$

If the sun rotated this angle in 49.2 hours, then the rotation period is:

$$\begin{aligned}T &= \frac{360^\circ}{19.8^\circ} \cdot 49.2 \text{ h} = 894.6 \text{ h} \\ &= \frac{894.6 \text{ h}}{24 \frac{\text{h}}{\text{d}}} \cong 37.3 \text{ d}\end{aligned}$$

This is too long, but (as I warned), these numbers were not real.