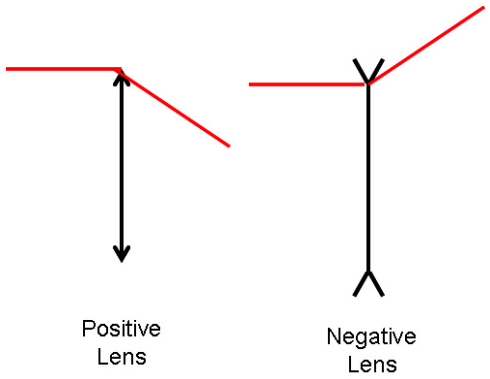
SIMG-215 Solution Set #1 Due 3 October 2006 (T)

These problems illustrate the imaging properties of lenses and use the two image equations that were mentioned in class:

$$\begin{aligned} \frac{1}{z_1} + \frac{1}{z_2} &= \frac{1}{f} \\ M_t &= -\frac{z_2}{z_1} \end{aligned}$$

Characterize the images (location and magnification) obtained using single thin lenses under the following conditions.
Sketch the systems showing the lens, focal points, object, and image. Use "arrows" to represent the lenses: a 'normal' arrow for a positive lens and an arrow with the heads 'reversed' for a negative lens:



Black Arrows = Lens, Red Lines = Rays

Algebraic Solution:

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f} \Longrightarrow \frac{1}{z_2} = \frac{1}{f} - \frac{1}{z_1}$$

$$\Longrightarrow \boxed{z_2 = \frac{1}{\left(\frac{1}{f} - \frac{1}{z_1}\right)}}$$

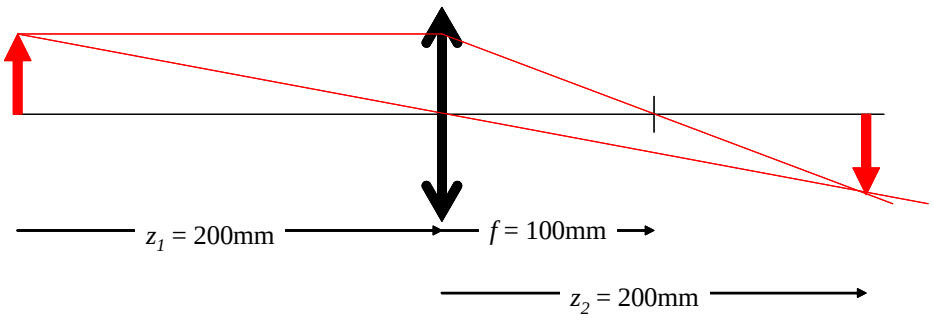
1. $f = +100\,\text{mm}$, $z_1 = +200\,\text{mm}$

numerical solution:

$$\begin{aligned}
\frac{1}{z_1} &= \frac{1}{+200\,\text{mm}} \\
\frac{1}{f} &= \frac{1}{+100\,\text{mm}} = \frac{2}{+200\,\text{mm}} \\
&\Longrightarrow \quad z_2 = \frac{1}{\left(\frac{1}{f} - \frac{1}{z_1}\right)} = \frac{1}{\left(\frac{2}{+200\,\text{mm}} - \frac{1}{+200\,\text{mm}}\right)} = \frac{1}{\left(\frac{1}{+200\,\text{mm}}\right)} = 200\,\text{mm} \\
M_T &= -\frac{z_2}{z_1} = -\frac{200\,\text{mm}}{200\,\text{mm}} = -1\text{: } \textit{image is real, inverted, and same size}
\end{aligned}$$

This is the "equal conjugates" solution

graphical solution:

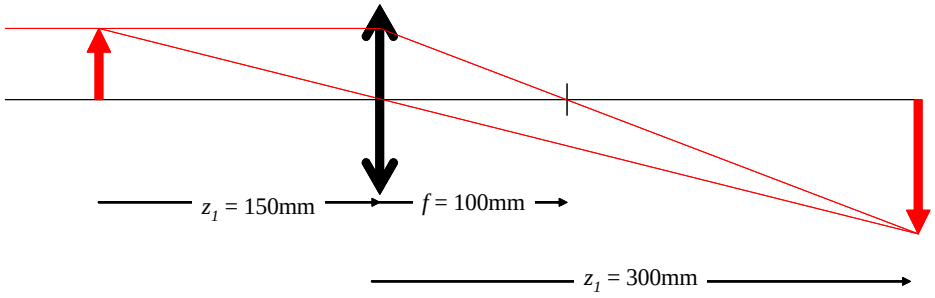


2. $f = +100\,\text{mm}$, $z_1 = +150\,\text{mm}$

numerical solution:

$$\begin{aligned}
\frac{1}{z_1} &= \frac{1}{+150\,\text{mm}} = \frac{2}{+300\,\text{mm}} \\
\frac{1}{f} &= \frac{1}{+100\,\text{mm}} = \frac{3}{+300\,\text{mm}} \\
&\Longrightarrow \quad z_2 = \frac{1}{\left(\frac{3}{+300\,\text{mm}} - \frac{2}{+300\,\text{mm}}\right)} = \frac{1}{\left(\frac{1}{+300\,\text{mm}}\right)} = 300\,\text{mm} \\
M_T &= -\frac{z_2}{z_1} = -\frac{300\,\text{mm}}{150\,\text{mm}} = -2\text{: } \textit{image is real, inverted, and twice as large}
\end{aligned}$$

graphical solution:

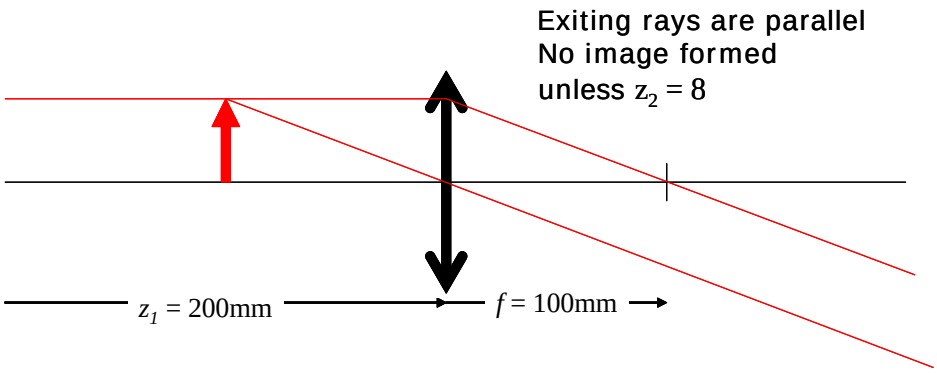


3. $f = +100\,\text{mm}$, $z_1 = +100\,\text{mm}$

numerical solution:

$$\begin{aligned}
\frac{1}{z_1} &= \frac{1}{+100\,\text{mm}} \\
\frac{1}{f} &= \frac{1}{+100\,\text{mm}} \\
&\Longrightarrow \quad z_2 = \frac{1}{\left(\frac{1}{+100\,\text{mm}} - \frac{1}{+100\,\text{mm}}\right)} = \frac{1}{0} = \infty \ \textit{(indeterminate large number)} \\
M_T &= -\frac{z_2}{z_1} = -\frac{\infty}{100\,\text{mm}} = \infty\textit{: image is real, inverted, and indeterminate (“infinite”) size}
\end{aligned}$$

graphical solution:



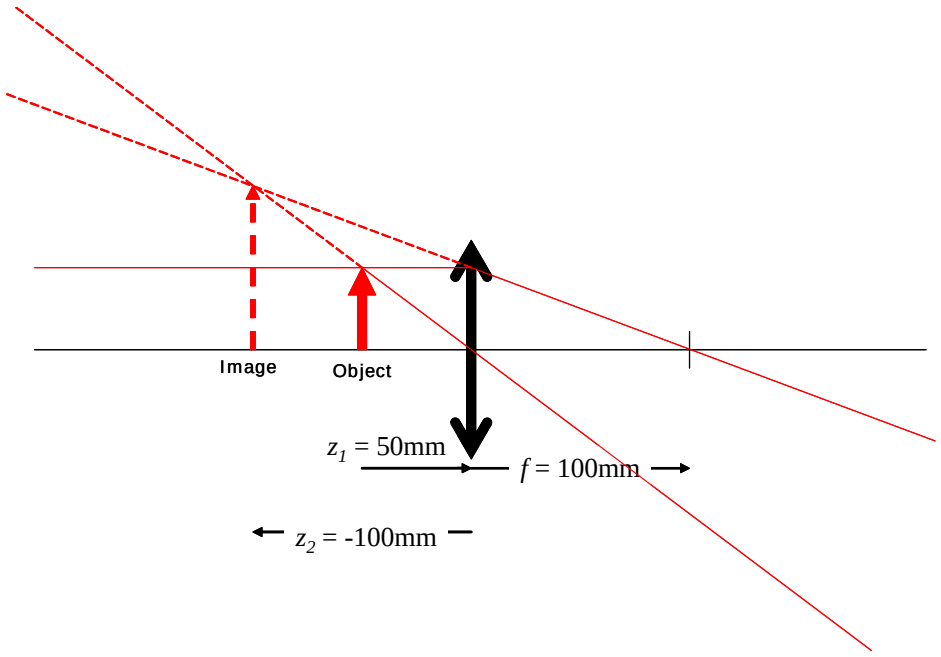
4. $f = +100\,\text{mm}$, $z_1 = +50\,\text{mm}$

numerical solution:

$$\begin{aligned}
\frac{1}{z_1} &= \frac{1}{+50\,\text{mm}} = \frac{2}{+100\,\text{mm}} \\
\frac{1}{f} &= \frac{1}{+100\,\text{mm}} \\
&\Longrightarrow \quad z_2 = \frac{1}{\left(\frac{1}{+100\,\text{mm}} - \frac{2}{+100\,\text{mm}}\right)} = \frac{1}{\left(-\frac{1}{100\,\text{mm}}\right)} = -100\,\text{mm} \\
M_T &= -\frac{z_2}{z_1} = -\frac{-100\,\text{mm}}{50\,\text{mm}} = +2\text{: } \textit{image is virtual, upright, and twice as large}
\end{aligned}$$

This is the "magnifier" or "magnifying glass"

graphical solution:

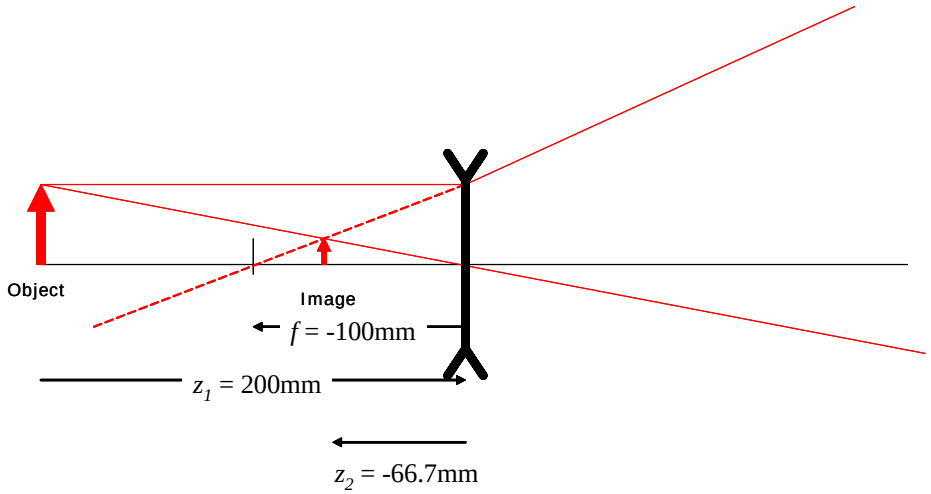


5. $f = -100\,\text{mm}$, $z_1 = +200\,\text{mm}$

numerical solution:

$$\begin{aligned}
\frac{1}{z_1} &= \frac{1}{+200\,\text{mm}} \\
\frac{1}{f} &= \frac{1}{-100\,\text{mm}} = -\frac{2}{200\,\text{mm}} \\
&\Longrightarrow \quad z_2 = \frac{1}{\left(-\frac{2}{200\,\text{mm}} - \frac{1}{+200\,\text{mm}}\right)} = \frac{1}{\left(-\frac{3}{200\,\text{mm}}\right)} = -\frac{200}{3}\,\text{mm} \cong -66.7\,\text{mm} \\
M_T &= -\frac{z_2}{z_1} = -\frac{-\frac{200}{3}\,\text{mm}}{+200\,\text{mm}} = +\frac{1}{3}\text{: } \textit{image is virtual, upright, and smaller}
\end{aligned}$$

graphical solution:

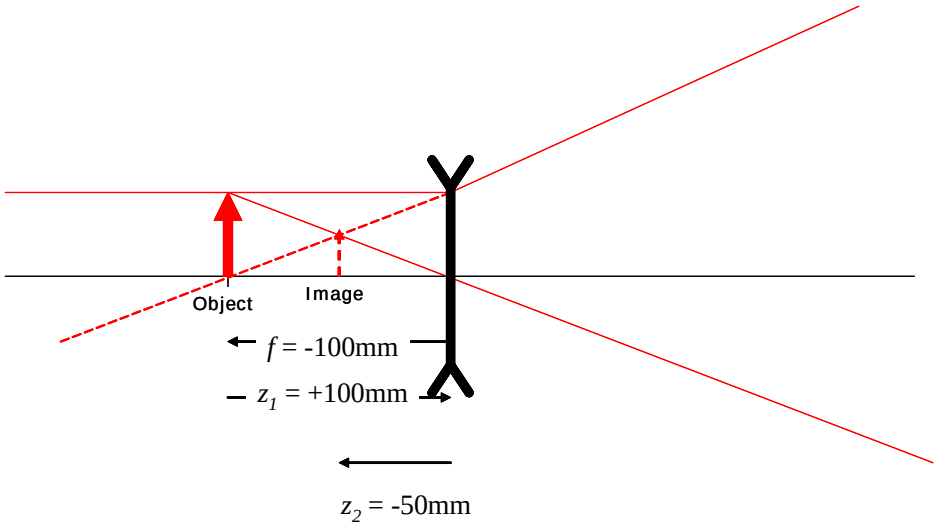


6. $f = -100\,\text{mm}$, $z_1 = +100\,\text{mm}$

numerical solution:

$$\begin{aligned}
\frac{1}{z_1} &= \frac{1}{+100\,\text{mm}} \\
\frac{1}{f} &= \frac{1}{-100\,\text{mm}} \\
&\Longrightarrow \quad z_2 = \frac{1}{\left(-\frac{1}{100\,\text{mm}} - \frac{1}{+100\,\text{mm}}\right)} = \frac{1}{\left(-\frac{2}{100\,\text{mm}}\right)} = -\frac{100}{2}\,\text{mm} = -50\,\text{mm} \\
M_T &= -\frac{z_2}{z_1} = -\frac{-50\,\text{mm}}{+100\,\text{mm}} = +\frac{1}{2}\text{: } \textit{image is virtual, upright, and smaller}
\end{aligned}$$

graphical solution:



7. $f = -100\,\text{mm}$, $z_1 = +50\,\text{mm}$

numerical solution:

$$\begin{aligned}
\frac{1}{z_1} &= \frac{1}{+50\,\text{mm}} = +\frac{2}{100\,\text{mm}} \\
\frac{1}{f} &= \frac{1}{-100\,\text{mm}} \\
&\Longrightarrow \quad z_2 = \frac{1}{\left(-\frac{1}{100\,\text{mm}} - \frac{2}{100\,\text{mm}}\right)} = \frac{1}{\left(-\frac{3}{100\,\text{mm}}\right)} = -\frac{100}{3}\,\text{mm} \cong -33.3\,\text{mm} \\
M_T &= -\frac{z_2}{z_1} = -\frac{-\frac{100}{3}\,\text{mm}}{+50\,\text{mm}} = +\frac{2}{3}\text{: } \textit{image is virtual, upright, and smaller}
\end{aligned}$$

graphical solution: