These problems illustrate the imaging properties of lenses and use the two image equations that were mentioned in class:

\[ \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f} \]

\[ M_t = -\frac{z_2}{z_1} \]

Characterize the images (location and magnification) obtained using single thin lenses under the following conditions.

Sketch the systems showing the lens, focal points, object, and image. Use “arrows” to represent the lenses: a ‘normal’ arrow for a positive lens and an arrow with the heads ‘reversed’ for a negative lens:

\[ \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f} \implies \frac{1}{z_2} = \frac{1}{f} - \frac{1}{z_1} \]

\[ \implies z_2 = \frac{1}{\left(\frac{1}{f} - \frac{1}{z_1}\right)} \]
1. \( f = +100 \text{ mm}, \ z_1 = +200 \text{ mm} \)

\[ \frac{1}{z_1} = \frac{1}{+200 \text{ mm}} \]
\[ \frac{1}{f} = \frac{1}{+100 \text{ mm}} = -\frac{2}{+200 \text{ mm}} \]

\[ \Rightarrow \quad z_2 = \frac{1}{\frac{1}{f} - \frac{1}{z_1}} = \frac{1}{\frac{2}{+200 \text{ mm}} - \frac{1}{+200 \text{ mm}}} = \frac{1}{\left(-\frac{1}{+200 \text{ mm}}\right)} = 200 \text{ mm} \]

\[ M_T = -\frac{z_2}{z_1} = -\frac{200 \text{ mm}}{200 \text{ mm}} = -1: \text{ image is real, inverted, and same size} \]

This is the “equal conjugates” solution

\[ z_1 = 200 \text{ mm} \quad f = 100 \text{ mm} \]

\[ z_2 = 200 \text{ mm} \]
2. \( f = +100 \text{ mm}, \ z_1 = +150 \text{ mm} \)

**numerical solution:**

\[
\frac{1}{z_1} = \frac{1}{+150 \text{ mm}} = \frac{2}{+300 \text{ mm}} \\
\frac{1}{f} = \frac{1}{+100 \text{ mm}} = \frac{3}{+300 \text{ mm}} \\
\Rightarrow z_2 = \frac{1}{\left(\frac{3}{+300 \text{ mm}} - \frac{2}{+300 \text{ mm}}\right)} = \frac{1}{\frac{1}{+300 \text{ mm}}} = 300 \text{ mm} \\
\]

\[ M_T = -\frac{z_2}{z_1} = -\frac{300 \text{ mm}}{150 \text{ mm}} = -2: \text{ image is real, inverted, and twice as large} \]

**graphical solution:**

![Graphical representation of the solution](image-url)
3. \( f = +100 \text{ mm}, \ z_1 = +100 \text{ mm} \)

**Numerical Solution:**

\[
\frac{1}{z_1} = \frac{1}{+100 \text{ mm}}
\]

\[
\frac{1}{f} = \frac{1}{+100 \text{ mm}}
\]

\[\implies z_2 = \frac{1}{\left(\frac{1}{+100 \text{ mm}} - \frac{1}{+100 \text{ mm}}\right)} = \frac{1}{0} = \infty \text{ (indeterminate large number)}\]

\[M_T = -\frac{z_2}{z_1} = -\frac{\infty}{100 \text{ mm}} = \infty: \text{ image is real, inverted, and indeterminate ("infinite") size}\]

**Graphical Solution:**

Exiting rays are parallel
No image formed
unless \( z_2 = 8 \)

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\( z_1 = 200 \text{ mm} \quad f = 100 \text{ mm} \)
4. $f = +100\text{ mm}, \ z_1 = +50\text{ mm}$

**numerical solution:**

$$\frac{1}{z_1} = \frac{1}{+50\text{ mm}} = \frac{2}{+100\text{ mm}}$$

$$\frac{1}{f} = \frac{1}{+100\text{ mm}}$$

$$\Rightarrow \ z_2 = \frac{1}{\left(\frac{1}{+100\text{ mm}} - \frac{2}{+100\text{ mm}}\right)} = \frac{1}{(-\frac{1}{100\text{ mm}})} = -100\text{ mm}$$

$$M_T = -\frac{z_2}{z_1} = -\frac{-100\text{ mm}}{+50\text{ mm}} = +2: \text{ image is virtual, upright, and twice as large}$$

*This is the “magnifier” or “magnifying glass”*

**graphical solution:**

![Graphical solution image showing the magnification of an object with a magnifier](image-url)
5. \( f = -100 \text{ mm}, \ z_1 = +200 \text{ mm} \)

**numerical solution:**

\[
\frac{1}{z_1} = \frac{1}{+200 \text{ mm}} \quad \frac{1}{f} = \frac{1}{-100 \text{ mm}} = -\frac{2}{200 \text{ mm}}
\]

\[\Rightarrow z_2 = \left( -\frac{2}{200 \text{ mm}} - \frac{1}{+200 \text{ mm}} \right) = \left( -\frac{3}{200 \text{ mm}} \right) = -\frac{200}{3} \text{ mm} \approx -66.7 \text{ mm} \]

\[M_T = -\frac{z_2}{z_1} = -\frac{-\frac{200}{3} \text{ mm}}{+200 \text{ mm}} = +\frac{1}{3}: \text{ image is virtual, upright, and smaller} \]

**graphical solution:**

![Diagram showing the object, image, and focal length with distances -200 mm and 200 mm, and the image position at -66.7 mm]
6. \( f = -100 \text{ mm}, \ z_1 = +100 \text{ mm} \)

numerical solution:

\[
\frac{1}{z_1} = \frac{1}{+100 \text{ mm}} \quad \frac{1}{f} = \frac{1}{-100 \text{ mm}}
\]

\[\Rightarrow \quad z_2 = \frac{1}{(-\frac{1}{100 \text{ mm}} - \frac{1}{+100 \text{ mm}})} = \frac{1}{(-\frac{2}{100 \text{ mm}})} = -\frac{100}{2} \text{ mm} = -50 \text{ mm} \]

\[M_T = -\frac{z_2}{z_1} = -\frac{-50 \text{ mm}}{+100 \text{ mm}} = +\frac{1}{2} : \text{image is virtual, upright, and smaller} \]

graphical solution:
7. \( f = -100 \text{ mm}, \ z_1 = +50 \text{ mm} \)

**Numerical solution:**

\[
\frac{1}{z_1} = \frac{1}{+50 \text{ mm}} = +\frac{2}{100 \text{ mm}}
\]

\[
\frac{1}{f} = \frac{1}{-100 \text{ mm}}
\]

\[
\Rightarrow z_2 = \frac{1}{\left(-\frac{1}{100 \text{ mm}} - \frac{2}{100 \text{ mm}}\right)} = \frac{1}{\left(-\frac{3}{100 \text{ mm}}\right)} = -\frac{100}{3} \text{ mm} \approx -33.3 \text{ mm}
\]

\[
M_T = -\frac{z_2}{z_1} = -\frac{-\frac{100}{3} \text{ mm}}{+50 \text{ mm}} = +\frac{2}{3}: \text{ image is virtual, upright, and smaller}
\]

**Graphical solution:**

![Graphical Solution](image-url)