SIMG-215-20061: LABORATORY #1

1 Test of Geometric Laws of Imaging

1.1 Objective:

The purpose of this experiment is to explore the geometric laws of image formation with two types of imaging system to determine whether the geometric law of magnification is a useful descriptor of the resulting images. The two types of imaging are (1) casting shadows of objects and (2) forming images with pinholes. The first is analogous to the standard type of medical X-ray image, where body structures are "projected" onto the sensor, while the second type has been used for many types of imaging applications in medicine and other discipliens

1.2 Materials:

1. your hand
2. 9 sides of white paper (5 sheets of paper)
3. pen or pencil
4. protractor to measure angles
5. meter stick or ruler
6. Light source, such as a desk lamp or flashlight
7. the Sun
8. Computer and monitor
9. Sheet of opaque paper or aluminum foil

1.3 Experimental Procedure 1: Shadows

Tape a piece of paper to a wall or place on a table or the floor. Position the nearby artificial light source (#1) such that it is a little farther than an arms' length ($B \cong 4\,\mathrm{ft} \cong 1.3\,\mathrm{m}$) from the paper, as illustrated in Figure 1. Now trace the outline that the shadow of your hand makes at a distance from the paper of $50\,\mathrm{mm} \cong 2\,\mathrm{in}$, $150\,\mathrm{mm} \cong 6\,\mathrm{in}$, and $300\,\mathrm{mm} \cong 12\,\mathrm{in}$. If the shadow of your whole hand won't fit on the paper, then use one or two fingers instead. Use one side of white paper for each shadow tracing (shadowgram).

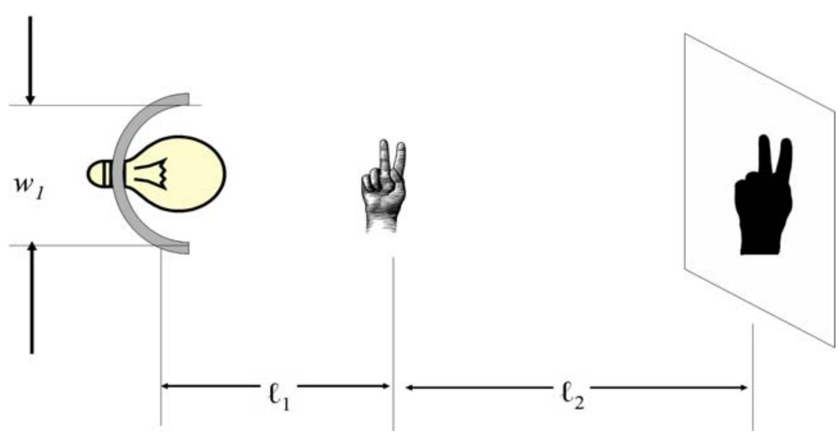


Figure 1: Formation of a "shadowgram," showing the distances to be measured.

Indicate on each shadowgram whether the shadow border is sharp or fuzzy, as well as the relative depth of the shadow (that is, how dark is the shadow?). You can do this either by annotation or by sheer drawing skill. Give the shadow an overall "shadow image quality" rating, 0 to 10.

Label each shadowgram with:

(A) the source size w_1 (this is the approximate width of the light source, which may be the lamp filament, if the lamp glass is transparent, or the diameter of the glass bulb, if the glass is frosted)

(B) the distance ℓ_1 from source to object

(C) the distance ℓ_2 from object to observation plane

(D) the "hand-to-paper distance," and

(E) a subjective quality rating.

The source size can be described both in *linear* units (e.g., mm) and *angular* units (angular degrees °). Estimate the angular source size θ in angular units of degrees. You can do this with a protractor, or you can use the values of w_1, ℓ_1, and ℓ_2. The angular size of the source as seen from the observation plane is proportional to the ratio of the diameter to the distance, which gives an approximate angle in *radians.* There are 2π radians in the 360° circumference of a circle, which means that the angular size of the source in degrees is:

$$\theta\,[^\circ] \cong \frac{w_1}{\ell_1+\ell_2} \times \frac{360^\circ}{2\pi} \cong \frac{w_1}{\ell_1+\ell_2} \times \frac{180^\circ}{3.14159} \cong \frac{w_1}{\ell_1+\ell_2} \times 57.3^\circ$$

(this equation is accurate to better than 2% error if $0^\circ \leq \theta \leq 45^\circ$).

In all cases, make a subjective judgment of the *quality* of the shadowgram image on the observation plane as a number between 0 and 9 using the "sharpness scale" shown below:

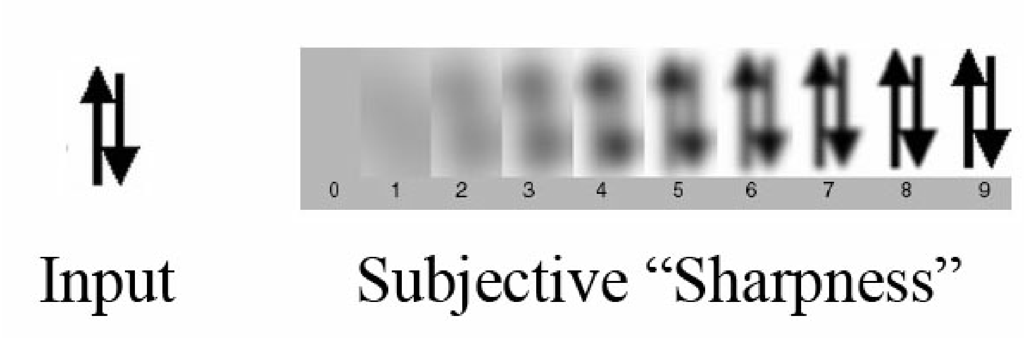


Figure 2: Subjective scale of image "sharpness"

You may estimate fractional values of sharpness if you feel the image is between two numbers. For example, an image that looks like it is between 6 and 7 might be estimated to have a sharpness of 6.5 .

Now move the light source farther away ($\ell_1 + \ell_2 \cong 2.5\,\text{m} \leq 8\,\text{ft}$) from the paper (ceiling height), and repeat the previous steps.

Repeat once more, this time using the Sun as your light source. DO NOT LOOK at the Sun. For your data table (see below), use the following: the Sun is at a distance of about $\ell_1 + \ell_2 \cong 1.5 \times 10^8\,\text{km} \cong 93,000,000\,\text{mi}$, and has a diameter of $w_1 \cong 1.4 \times 10^6\,\text{km} \cong 866,000\,\text{mi}$. The angular size of the Sun as seen from Earth is:

$$\theta\,[^\circ] \cong \frac{1.4 \times 10^6\,\text{km}}{1.5 \times 10^8\,\text{km}} \times \frac{180^\circ}{\pi} \cong 0.5^\circ$$

At the end you should have 9 shadowgrams, each labeled with its light source, the source-to-paper distance, and the hand-to-paper distance. You should have enough data to construct the basic data table:

Light Source	w_1	ℓ_1	ℓ_2	Angular size θ	Quality
Light bulb			50 mm		
Light bulb			150 mm		
Light bulb			300 mm		
Light bulb			50 mm		
Light bulb			150 mm		
Light bulb			300 mm		
Sun	1.4×10^6 km	1.5×10^8 km	50 mm	0.5°	
Sun	1.4×10^6 km	1.5×10^8 km	150 mm	0.5°	
Sun	1.4×10^6 km	1.5×10^8 km	300 mm	0.5°	

1.4 Experimental Procedure 2: Imaging with Pinholes

The pinhole imaging system is fundamentally different from the "shadowgram" setup. In this case, the object to be imaged is at the "input" side of the system, instead of in the middle in the shadowgram system.

The imaging system is shown in Figure 3. The original object you will use is a test pattern on the computer monitor. Measure the width w_1 of this test pattern as the distance between the centers of the vertical bars.

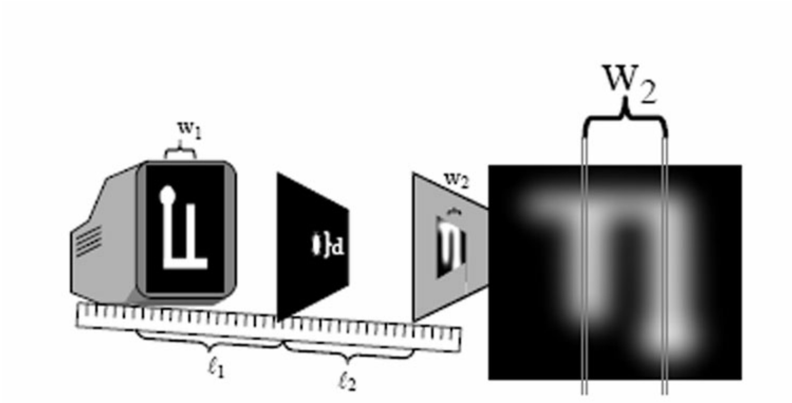


Figure 3: Formation of a "Pinhole" image

Select (or make) a pinhole and measure its diameter. The distance ℓ_1 is measured from the object to the pinhole, Select locations ℓ_1 and ℓ_2 and measure the corresponding distance between the vertical bars in the image; this is w_2.

Select additional pairs of positions ℓ_1 and ℓ_2 and repeat the same measurements. Use the widest range of locations that your experimental equipment will allow. Record your data in the data table.

The difference (or the *deviation*) of the measurement is $\Delta = w_p - w_2$, which is measured in the same units as w_p and w_2 (typically millimeters); it measures the "disagreement" between the theory and the experiment. Calculate Δ for each value of your experimental measurements, and include in your tables.

Repeat the experiment for all pinholes.

1.4.1 Data Tables:

I. pinhole diameter $d = 2.0\,\text{mm}$, $w_1 =$ ______ mm

ℓ_1 [mm]	ℓ_2 [mm]	w_2 [mm]	Quality	$w_p = \frac{\ell_2}{\ell_1} w_1$ [mm]	$\Delta = w_2 - w_p$

II. pinhole diameter $d = 5.0\,\text{mm}$, $w_1 =$ ______ mm

ℓ_1 [mm]	ℓ_2 [mm]	w_2 [mm]	Quality	$w_p = \frac{\ell_2}{\ell_1} w_1$ [mm]	$\Delta = w_2 - w_p$

III. pinhole diameter $d = 10.0\,\text{mm}$, $w_1 =$ ______ mm

ℓ_1 [mm]	ℓ_2 [mm]	w_2 [mm]	Quality	$w_p = \frac{\ell_2}{\ell_1} w_1$ [mm]	$\Delta = w_2 - w_p$

IV. pinhole diameter $d = 20.0\,\text{mm}$, $w_1 =$ ______ mm

ℓ_1 [mm]	ℓ_2 [mm]	w_2 [mm]	Quality	$w_p = \frac{\ell_2}{\ell_1} w_1$ [mm]	$\Delta = w_2 - w_p$

1.5 Data Analysis:

1.5.1 Magnification:

The geometric scaling law for a pinhole imaging system is:

$$w_1\,[\mathrm{mm}] \cdot \frac{\ell_2\,[\mathrm{mm}]}{\ell_1\,[\mathrm{mm}]} = w_2\,[\mathrm{mm}]$$

Use your values of w_1, ℓ_1, and ℓ_2, to calculate an estimate of w_2; call this w_p. Plot your values of w_p versus your measured values of w_2, i.e., for each value of w_2 on the x-axis, plot the value of w_p on the y-axis. Ideally the values of w_p and w_2 should be the same. Note that the equation does *not* contain the diameter d of the pinhole, which means that the same results are predicted for all pinholes regardless of diameter.

1.5.2 Resolution:

The geometric law of resolution predicts that the *sharpness* of the image depends only on the ratio of the diameter d of the pinhole and the linear dimension w_1 of the object:

$$\textit{sharpness} \propto \frac{w_1}{d}$$

In words, the larger the pinhole, the poorer the sharpness. You can test this by plotting your estimated sharpness values on the y-axis for the pinhole diameter d on the x-axis. The data should cluster around each pinhole diameter and be significantly different for different diameters.

1.5.3 Discussion:

Part 1: The geometric law of magnification predicts that w_p and w_1 should be identical and that the values of w_p should not depend on the diameter d of the pinhole.

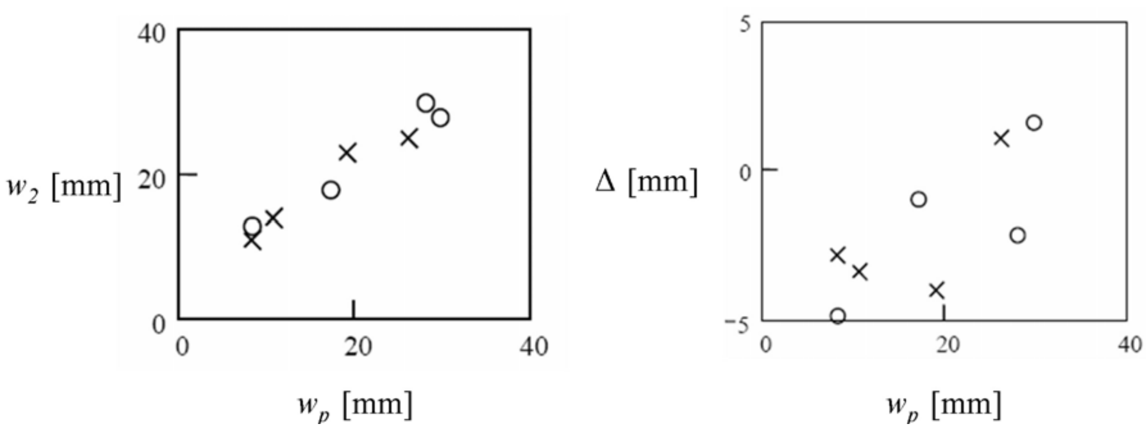
There are two possible reasons why Δ may not be zero. The first is that there will be errors in the measurments; the second is that the equation for geometric magnification for a pinhole system may not be correct. Experimental measurements always have some amount of error, and the job of the scientist is to estimate the amount of error in the experiment. Based on your estimate of the amount of experimental error that is likely to be in your measurements, you should be able to reach one of the following conclusions:

A. The geometric law of magnification is a useful description of pinhole imaging.

B. The geometric law of magnification does not adequately describe pinhole imaging.

1.6 Examples of Possible Results

The data are plotted as "X" for a pinhole with diameter $d_X = 2\,\mathrm{mm}$ and with an "O" for $d_O = 10\,\mathrm{mm}$.



Sample graphs of data