Imaging with Lenses

- Distance from Object to Lens is $z_1$
- Distance from Lens to Image is $z_2$
- Relate $f$ to $z_1$ and $z_2$ through the Imaging Equation

Imaging with Positive Lens

$z_1 > f$
$z_2 < 0$

Image Object
Observations for Positive Lens

<table>
<thead>
<tr>
<th>Object Distance $z_1$</th>
<th>Image Distance $z_2$</th>
<th>Imaging Equation for Lens</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\infty$</td>
<td>$f &gt; 0$ (real image)</td>
<td>$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$</td>
</tr>
<tr>
<td>$&gt; f$</td>
<td>$&gt; f$ (real image)</td>
<td></td>
</tr>
<tr>
<td>$f &gt; 0$</td>
<td>$\infty$ (&quot;real image&quot;)</td>
<td></td>
</tr>
<tr>
<td>$&lt; f$</td>
<td>$&lt; 0$ (virtual image)</td>
<td></td>
</tr>
</tbody>
</table>

Example:

■ Where is the image if $z_1 = 2f$?

$$\frac{1}{2f} + \frac{1}{z_2} = \frac{1}{f} = \frac{2}{2f}$$

$$\frac{1}{z_1} = \frac{2}{2f} \quad \frac{1}{z_2} = \frac{1}{2f}$$

$$z_2 = 2f = z_1$$

Image Distance = Object Distance ("Equal Conjugates")

Diagram of Lens at Equal Conjugates

■ Cast two rays from tip of object
  1. parallel to axis of symmetry, refracts through "rear" focal point
  2. through center of lens, not refracted
  3. through "front" focal point, refracts parallel to axis

■ These rays "converge" at the image point
Imaging at Equal Conjugates

- Image size is identical to object size, but "upside down" ("inverted")

\[
\theta = \frac{\ell_1}{z_1} = -\frac{\ell_2}{z_2}
\]

\[
M = \frac{z_2}{\ell_1} = \frac{z_2}{z_1}
\]

Image Magnification

- Ratio of the size of the output image compared to the input object?

\[
\theta = \frac{\ell_1}{z_1} = -\frac{\ell_2}{z_2}
\]

\[
M = \frac{z_2}{\ell_1} = \frac{z_2}{z_1}
\]

Diagram of Lens with \(2f > z > f\)

- Example: \(z = 1.5f\)

\[
\frac{1}{3} \cdot \frac{1}{z} = \frac{1}{f} \Rightarrow z_1 = 3f \Rightarrow M = \frac{z_2}{z_1} = -\frac{3f}{\left(\frac{3}{2}\right)} = -2
\]
Diagram of Lens with $z_1 < f$

- Virtual Image

Tracing Rays for Negative Lens

- Image:
  - $f < 0$
  - Image is Upright, Virtual and Smaller ("minified")

Imaging with a Negative Lens

- Focal length $f$ is negative: $f = -|f|$.
- Place object at distance $z_1 = 2|f|$.

\[
\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f} = -\frac{1}{|f|} \\
z_1 = 2|f| \Rightarrow z_2 = -\frac{2|f|}{3} \\
M_f = -\frac{z_2}{z_1} = -\frac{-\frac{2|f|}{3}}{2|f|} = \frac{1}{3}
\]
**Imaging Chain Link #4**

**Energy Sensors**
- Convert energy propagated from object to a measurable physical parameter
  - e.g., number of silver atoms or number of electrons

**Image Capture = Detection = Sensing**
- Image captured by sensor, e.g., photographic emulsion, CCD
  - Emulsions must be processed to yield a permanent visible image
  - CCDs must be read out to produce an array of numbers that may be interpreted as an image

**Most Common Sensors**
- Human Eye
- Photographic Emulsion
  - film
  - plates
- Electronic Sensors
  - "Charge-Coupled Devices" = CCDs
  - "Complementary Metal-Oxide Semiconductors" = CMOS
  - "Charge-Injection Devices" = CIDs
Eye as Detector

- Eye includes its own lens
  - focuses light on retina ("sensor")
- When used with an optical device (e.g., telescope), must add yet another lens
  - redirect rays from primary optic
  - make them parallel ("collimated")
    - rays appear to come from "infinity" (infinite distance away)
  - reimaging is performed by "eyepiece"

Eye with Telescope

- Light entering eye is "collimated"

Photographic Emulsions

- More difficult to quantify
  - Light-sensitive "grains" of silver halide in the emulsion
  - Placed "randomly" in emulsion
    - "Random" sizes
  - "Large" grains are more sensitive (respond to few photons)
  - "Small" grains produce better resolution