R·I·T
2010 Imaging Science Ph.D. Comprehensive Examination
June 11, 2010
9:00AM to 1:00PM

IMPORTANT INSTRUCTIONS

You must complete two (2) of the three (3) questions given for each of the core graduate classes. The answer to each question should begin on a new piece of paper. While you are free to use as much paper as you would wish to answer each question, please only write on one side of each sheet of paper that you use. Be sure to write your provided identification letter (located below), the question number, and the page number for each answer in the upper right-hand corner of each sheet of paper that you use. When you hand in your exam answers, be certain to write your name and your provided identification letter on the supplied 5” x 8” note card and place this in the envelope located with the proctor.

ONLY HAND IN THE ANSWERS TO THE QUESTIONS THAT YOU WOULD LIKE EVALUATED

Identification Letter: ________________

THIS EXAM QUESTION SHEET MUST BE HANDED BACK TO THE PROCTOR UPON COMPLETION OF THE EXAM PERIOD
1. Fourier Methods in Imaging.

For the 2-D input object:

\[ f(x, y) = 1 \cdot x \cdot y \cdot \exp[i \cdot 0.1 \cdot \text{RECT}(x - 1, y)] \]

a. Sketch profiles of \( f(x, y) \) along BOTH the \( x \)- and \( y \)-axes: real part, imaginary part, magnitude, and phase.

b. The function \( f(x, y) \) is to be filtered by an ideal highpass filter that blocks a small region of the frequency domain near the origin. Derive expressions for the output \( g(x, y) \) and \( |g(x, y)|^2 \) – you may reasonable approximations if necessary.

c. Sketch profiles along the \( x \)- and \( y \)-axes of \( |g(x, y)|^2 \)

d. Describe an application for this filter.
2. Fourier Methods in Imaging.

A 2-D real-valued input object $f[x, y]$ has the form:

$$f[x, y] = \exp \left[ -i\pi \left( x - x_0 \right)^2 \right] \cdot \exp \left[ b_0 \cdot y \right] \cdot \text{STEP}[-y]$$

a. Find an expression for the impulse response $m[x, y]$ of the ideal matched filter for $f[x, y]$.

b. Sketch profiles of the impulse response of the ideal matched filter along the $x$ and $y$ axes.

c. Determine the output of this filter if the input function is:

$$g[x, y] = \exp \left[ -i\pi \left( \frac{x}{b_0} \right)^2 \right] \cdot \frac{1}{2} \exp \left[ b_0 \left( y - 2 \right) \right] \cdot \text{STEP}[-y]$$

Given a system with transfer function

\[ H[\xi] = \exp\left[+i\pi (1 - 2 \cdot RECT[\xi])\right] \]

a. Find an expression for and sketch the impulse response \( h[x] \).

b. Find an expression for and sketch the form of the output \( g[x] \) when \( f[x] = \cos[\pi x] - \cos[2\pi x] \)

c. Find an expression for and sketch the transfer function and the impulse response of the “inverse filter” for \( H[\xi] \).
4. Radiometry.

The joker is trying to avoid detection by Batman’s new remote controlled Bat Plane with night vision sensors. He has acquired a new laser with 3W of radiant power and a beam width of $1 \cdot 10^{-3}$ sr operating at 0.9 $\mu$m so Batman won’t know he is searching for the plane. He has managed to locate detectors every 500m from his laser beam along the road from town over which the Bat Plane approaches each night (see figure below) at 1000 m altitude. His sources tell him the plane has a diffuse reflectance of 5% and is about 5 m$^2$ in horizontal cross section. To warn him of the approach on a moonless night with high clouds (i.e. dark), he has his laser at an elevation angle of 20° and watches for a signal from his detectors which are 1 cm$^2$, have a quantum efficiency of 0.85, approximately 100 noise electrons, and integrate for about $\frac{1}{100}$ sec

![Figure of laser beam and detectors](image)

Will the joker carry out his nefarious plans for the evening, or will the Batman foil him once again? Potentially useful information:

- $h = 6.623 \cdot 10^{-34}$ joules $\cdot$ sec
- $c = 3 \cdot 10^8$ m $\cdot$ sec$^{-1}$
5. Radiometry.

I am trying to locate a rare sea snake that emerges at night and crawls on the sandy bottom of a turbid tropical lagoon. The snake is about a 25% reflector, and the sand about a 20% reflector. My first attempts using a co-located source and camera failed because I had too much reflected energy from the water. So I’ve devised a new plan as shown in the figure. My 1000 $Wsr^{-1}$ source is suspended off the back of my 7 $m$ long dingy, while I image vertically with a camera just below the surface. I assume that in using this design I will have negligible scatter from the small water volume above the bottom. I’ve measured the extinction coefficient of the water to be 0.1 $m^{-1}$. My camera has a $f/2$ lens with a 94% transmission. The detector elements are 12 $um$ on a side and have noise equivalent power of $1 \cdot 10^{-11} W$. If I’m in 7 $m$ of water, do I have a chance of differentiating a snake from the bottom?

(note the snake should be fully resolved at this distance, i.e., some pixels should be filled with the image of the snake)
6. Radiometry.

3) I am using a $\lambda = 500 nm$ pulsed laser ($2 \cdot 10^{-5} sec/pulse$) system to measure the dynamic stresses in a plastic material. The stresses cause a deformation of the plastic that produces small changes in the reflectivity of the surface. I have determined that the smallest stress levels of interest will induce reflectance factor changes of 0.5%. Given the design shown below, where I spread the 0.3 $W$ laser to fill 0.025 $sr$ using a lens with a transmission of 0.95 and illuminate and view roughly perpendicular to the approximately 10% reflecting surface, do I have a chance of detecting the change? My camera has an $f/4$ lens, 15 $\mu m$ on a side detectors, and a lens transmission on axis of 0.92.

Potentially useful information:

\[ h = 6.623 \cdot 10^{-34} \text{joules} \cdot \text{sec} \]

\[ c = 3 \cdot 10^8 m \cdot \text{sec}^{-1} \]

Adult human visual acuity is limited to ~20/10 on the Snellen acuity scale. Describe the Snellen scale and the specific reasons for the limit. For each reason, discuss the limiting value and whether there is a practical solution that could be implemented to overcome the limit.

Define the monocular Contrast Sensitivity Function.

a. Sketch the foveal, monocular CSF of an average adult observer under photopic conditions.

b. Describe in detail a method for measuring the CSF under these conditions.

c. Sketch the monocular CSF of an average adult observer for a target 15° to the left of center along the horizontal axis.

d. Describe in detail a method for measuring the CSF under these conditions.

e. How and why would it be different than the CSF in b. above.

f. Does it make a difference which eye you measure? Why?

The Gabor function below (a 1-dimensional $\sin$ windowed by a 2-dimensional Gaussian) is viewed foveally by an average adult observer. (It is viewed against a black background extending across the entire visual field.) The test pattern contains 15 cycles of the $\sin$ pattern and is 1 cm wide.

Although the observer is allowed to adjust the contrast of the Gabor patch to any value, it is found to be just at the threshold of detection, i.e., it can just be reliably distinguished from a Gaussian patch with the same mean luminance.

a. Determine the approximate distance the observer was from the test pattern.
b. Would your answer change if the Gabor were rotated $90^\circ$? Defend your answer.
c. Would your answer change if the Gabor were rotated $45^\circ$? Defend your answer.
d. Would your answer change if the Gabor was black & blue? Defend your answer.
e. Would your answer change if the Gabor was black & yellow? Defend your answer.

Consider following signal:

\[ f(t) = \sin \left( \frac{2\pi t}{T} \right), \quad 0 \leq t \leq T \]
\[ = 0 \quad \text{otherwise} \]

Will there be aliasing if you use a sampling interval = \( T/4 \)? Justify your answer.

A 64x64 digital image with 6 gray level has a histogram given by

<table>
<thead>
<tr>
<th>Gray level-i</th>
<th>No. of pixels with gray level-i</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>614</td>
</tr>
<tr>
<td>1</td>
<td>328</td>
</tr>
<tr>
<td>2</td>
<td>819</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
</tr>
<tr>
<td>4</td>
<td>1434</td>
</tr>
<tr>
<td>5</td>
<td>751</td>
</tr>
</tbody>
</table>

Develop the Look-up-table (LUT) to perform histogram equalization on this image.

A noise free one dimensional real signal has a one-sided () power spectrum that can be modeled by $P_f(\xi)$, where $\xi$ stands for spatial frequency in units of cycles/cm. The signal that has to undergo digital processing has additive noise, whose power spectrum can be modeled as $P_n(\xi)$.

$$P_f(\xi) = 4RECT\left[\frac{\xi - 5}{10}\right] ; \quad P_n(\xi) = 2RECT\left[\frac{\xi - 12}{8}\right]$$

a. Plot the signal and the noise power spectrum

b. We would like to filter the noise using any one of the following digital convolution kernels. Assume sampling interval of 0.02 cm for the spatial domain data and the kernel.

- Kernel-A = $[1/3 , 1/3 , 1/3]$
- Kernel-B = $[1/5 , 1/5 , 1/5 , 1/5 , 1/5]$
- Kernel-C = $[-1 , 2 , -1]$
- Kernel-D = $[+1 , -1]$

Which kernel will you use to get the best results? Justify your answer based of the knowledge of the transfer function of the kernel.
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13. Optics for Imaging.

A spherical lightbulb is made from a glass bead with inner and outer diameters $d_1$ and $d_2$, respectively, that are centered at the same point. The refractive index of the glass is $n_1$ and the outgoing rays are observed in air. The filament of the bulb is located at the center of the spheres and is sufficiently small so that it may be approximated as a 3-D Dirac delta function. The inner sphere is filled with a (hypothetical) gas with $n_{\text{gas}} = 1.1$. Light at all points on the surface of the inner sphere is scattered uniformly over $2\pi$ steradians in the outward direction. The bulb is viewed at a large enough distance so that the rays of light emerging from the outer sphere may be accurately modeled as traveling parallel paths.

a. Determine the ratio of the outer and inner diameters such that the light from the bulb appears to fill the entire cross-sectional area of the glass bead.

b. Describe the changes that occur (if any) if the light bulb is immersed in water ($n_{\text{water}} \approx \frac{4}{3}$)?

A 2-D transparency $f(x, y)$ is illuminated by a monochromatic plane wave with wavelength $\lambda_0$. The light through the transparency travels the distance $z_1$ in the Fresnel diffraction region to a lens with focal length $f_1 > 0$ and aperture diameter that is effectively “infinitely large.”

a. Derive expressions for the amplitude $g_1(x, y; z_1)$ and irradiance that would be observed at a distance $z_2 = f_1$ after the lens. You may delete any constant factors (magnitude and/or phase) from your answers. The expressions should depend upon the distance $z_1$.

b. Evaluate the irradiance for these cases:

   (a) $z_1 = 0$
   (b) $z_1 = f_1$
   (c) $z_1 = 2 \cdot f_1$
   (d) $z_1 = \infty$
15. **Optics for Imaging.**

An imaging system in air is constructed from three thin lenses:

<table>
<thead>
<tr>
<th>Lens</th>
<th>focal length</th>
<th>diameter</th>
<th>separation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1$</td>
<td>$f_1 = -100mm$</td>
<td>$d_1 = 25mm$</td>
<td>$t_{12} = 100mm$</td>
</tr>
<tr>
<td>$L_2$</td>
<td>$f_2 = +75mm$</td>
<td>$d_2 = 25mm$</td>
<td>$t_{23} = 100mm$</td>
</tr>
<tr>
<td>$L_3$</td>
<td>$f_3 = -100mm$</td>
<td>$d_3 = 25mm$</td>
<td></td>
</tr>
</tbody>
</table>

a. Determine the focal length of the system.

b. Locate the focal and principal points of the system and make a sketch of the system that includes them.

c. Determine which lens is the “aperture stop” of the system for an object at an infinite distance away; explain your reasoning.

d. Locate the entrance and exit pupils of the system and find their sizes.

e. Define the focal ratio of an optical system and calculate it for this example.

f. Locate the image of an object located at the point $O$ such that its distance from the object-space vertex is $\overline{OV} = 100mm$.

g. Determine the transverse and longitudinal magnifications of the image of the object with $\overline{OV} = 100mm$. Explain the algebraic signs of the two magnifications.

h. Remove the positive lens from the system while keeping the other lenses fixed in place. Repeat steps (a) - (g) for this system.
16. DIP.

What would a exam be like without a question involving our favorite image? [That is not the real question, so read on ...] There are two highlighted regions on the image of Lenna shown below.

The image was cropped two different times so that only these highlighted portions remained.

These two regions were then saved using the JPEG compression process in Adobe Photoshop with a quality factor of 3. Remember that Adobe Photoshop allows you to select
a “Quality” when saving a JPEG-compressed image that ranks from “Maximum (12)” to “Low (3)”. Both of these regions are 32x32 pixels in size. The area outlined in white resulted in a file that required 33,824 bytes to save while the area outlined in black resulted in a file with 32,711 bytes in it. These JPEG files are redisplayed below.

With the information given above and the images that are displayed here for your inspection, explain 1) why the cropped images resulted in files that required varying amounts of disk space, and 2) why the redisplayed images exhibit different levels of artifacts. Be specific in your answers with regards to particular portions of the JPEG compression process as well as your knowledge of the human visual system.
17. DIP.

Given the conditional probability \( p(x \mid \omega_i) \) where \( x \) represents an \( N \)-dimensional digital count vector and \( \omega_i \) represents a pixel type of interest, \( i \), derive the linear discriminant function \( g(x) \) that could be used for Gaussian maximum likelihood classification using the decision rule

\[
x \in \omega_i \text{ if } g_i(x) > g_j(x) \text{ for all } i \neq j
\]

and comment on the physical significance of each of the terms in the functional form that you have derived.
18. DIP.

Using the basic operators associated with binary morphological image processing, logic, and set theory:

⊕ dilation
⊗ erosion
⊙ opening
• closing

! negation
∪ union
∩ intersection
⊂ subset

design an approach for region filling of a provided binary boundary image, \( B \), where the boundary pixels have a digital count value of 1 and the non-boundary pixels have a digital count value of 0.

Let the input to a photon detector be represented by $X = q$ and the output by $Y$. When a photon enters the detector it generates $M$ output events, where $M$ is a random variable whose distribution is described by the table below. $X$ is a Poisson random variable with $E[X] = \bar{q}$ photons.

<table>
<thead>
<tr>
<th>$M$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(M)$</td>
<td>0.1</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Find an expression for the DQE in terms of $\bar{q}$.

Two discrete random processes \( x(n) \) and \( y(n) \) are related by

\[
y(n) = \sum_{k=-\infty}^{n} x(k)h(n-k)
\]

where

\[
h(n) = Ae^{-an} \text{step}(n) \quad \text{and} \quad H(\omega) = \sum_{n=-\infty}^{\infty} h(n)e^{-i\omega n} = \frac{A}{1 - e^{-a-i\omega}}
\]

In the following assume that \( x(n) \) is a white-noise sequence with zero mean and variance \( \sigma_x^2 \).

a. Find an expression for the autocorrelation function \( R_{yy}(m) \) in terms of \( m, A, a \) and \( \sigma_x \).

b. Find an expression for the mean-squared value of \( y(n) \) in terms of \( A, a \) and \( \sigma_x \).

A detector with a DQE of 0.5 is to be used to distinguish between two photon sources. Source $S_1$ emits photons at a rate of $\lambda_1 = 900$ photons/millisecond and source $S_2$ emits photons at a rate of $\lambda_2 = 1000$ photons/millisecond. The detector output count $y$ obtained by exposing it for $T$ milliseconds is to be compared to a decision threshold $b$. The decision rule is to decide $S_1$ if $y \leq b$ and decide $S_2$ if $y > b$. Calculate values for $T$ and $b$ such that the of a decision error is 0.1, assuming the two sources are equally likely. You may use the normal PDF approximation in your calculations.